How many marbles can fit under the Eiffel Tower?

Presented by:
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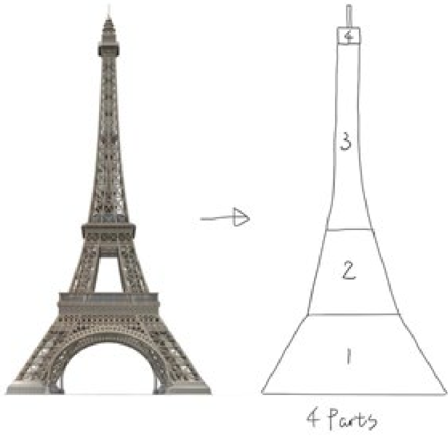
Brief



Main Steps:

1. Work out the total volume of the Eiffel Tower (V1)
2. Work out the volume of iron frame (V2)
3. Volume of space for the marbles (V3 = V2 – V1)
4. Work out the volume of a single marble (V4)
5. Number of marbles = V3 / V4 (Stacking efficiency)

For Step 1:

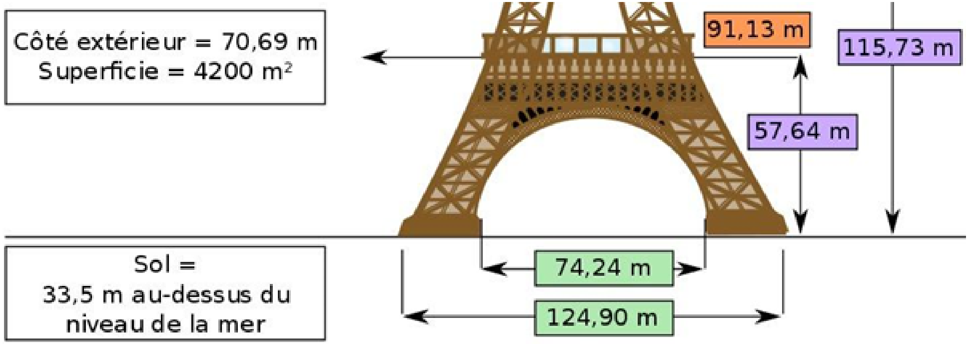
Section 1: Formula method of a truncated square pyramid

Section 2: Use of integration

Section 3: Two different methods & Comparison

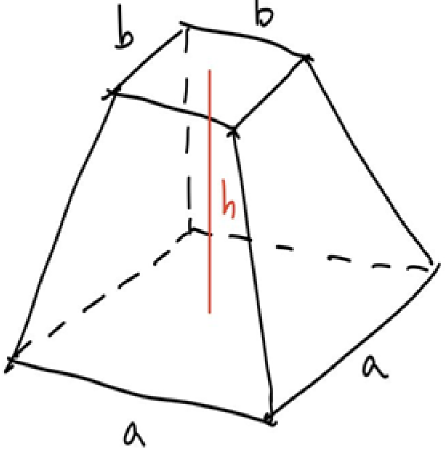
Section 4: Formula methods of cubes and cylinders

Molly - Layer one



Volume of Trapezoidal Footing ✓

Formula = $\frac{1}{3}(a^2 + ab + b^2)\,h$



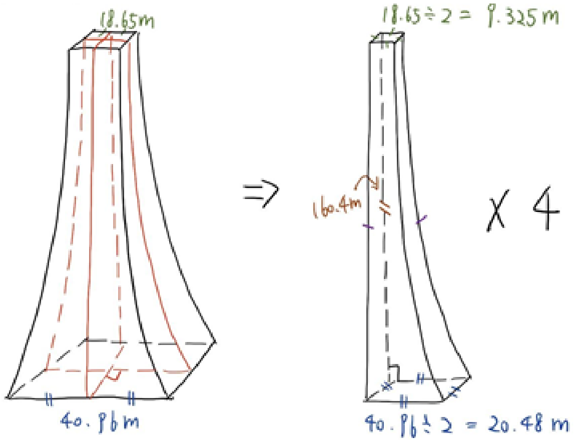
$$V = \frac{1}{3}\left(70.69^2 + 70.69 \times 124.9 + 124.9^2\right) \times 57.64$$

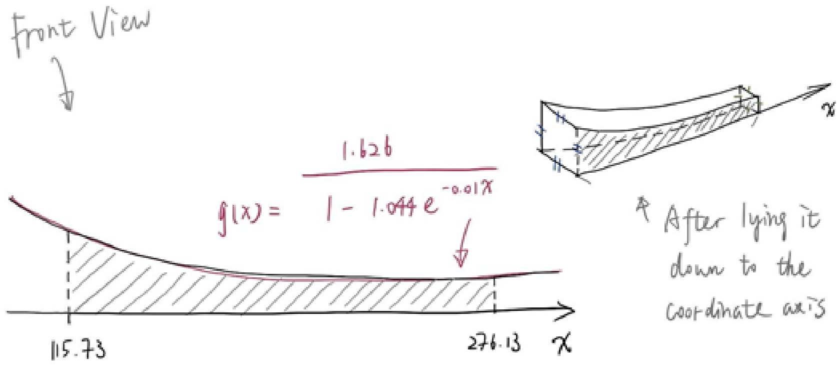
$$= \frac{1}{3} \times 29426.27 \times 57.64$$

$$= 565376.679 \text{ m}^3$$

Integration Steps

Step one:

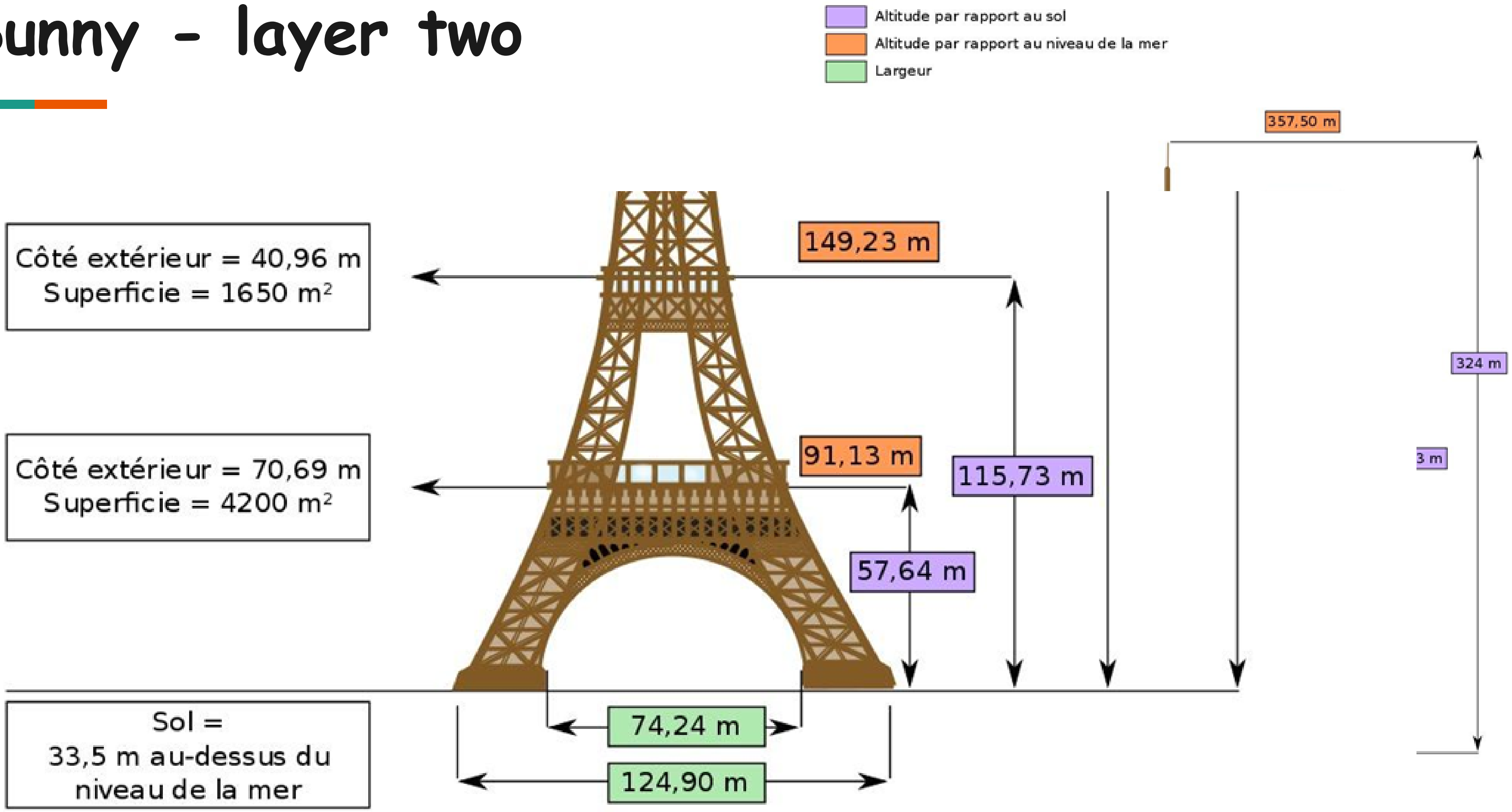


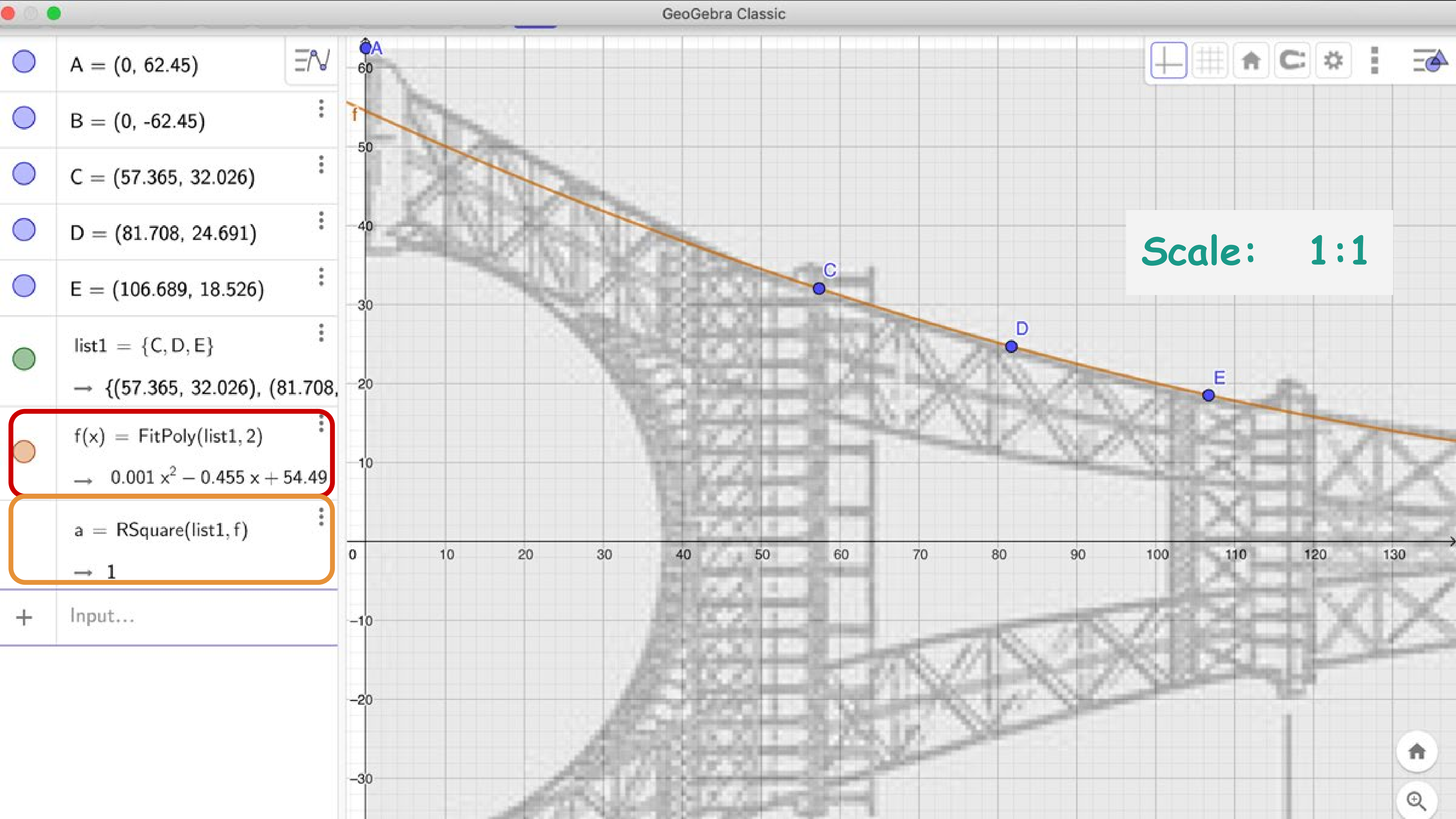


Step two:

rhttps://www.youtube.com/watch?v=4vLy5VoUcQE&t=213s

Sunny - layer two




GeoGebra Classic
A = (0, 62.45)
B = (0, -62.45)
C = (57.365, 32.026)
D = (81.708, 24.691)
E = (106.689, 18.526)
list1 = {C, D, E}
→ {(57.365, 32.026), (81.708,
f(x) = FitPoly(list1, 2)
→ 0.001 x² – 0.455 x + 54.49
a = RSquare(list1, f)
→ 1
Input...
Scale: 1:1
A
C
D
E
f
0
10
20
30
40
50
60
70
80
90
100
110
120
130
−10
−20
−30

Calculations

Volume of each identical part:

Layer 2 :

① $V = \int_{57.64}^{115.73} (f(x) - 0)^2 \, dx$

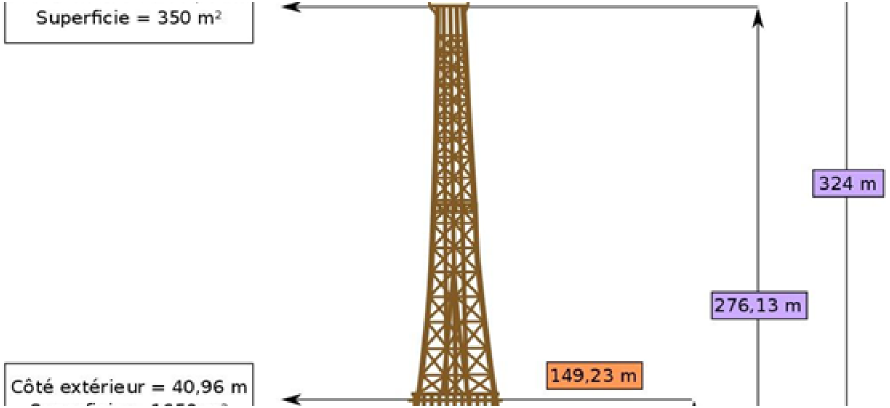
$= \int_{57.64}^{115.73} (0.001x^2 - 0.455x + 54.49 - 0)^2 \, dx$

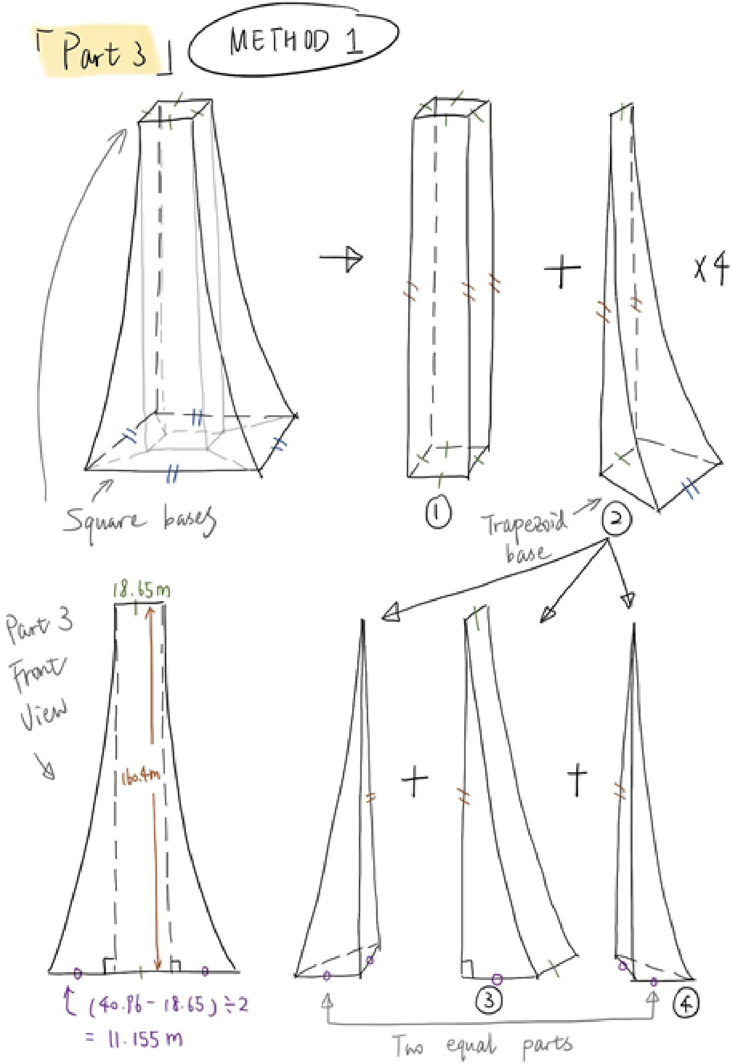
$= 31612.991 \ m^3$

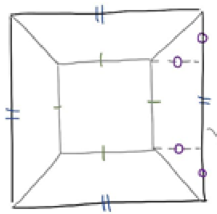
Final volume of layer 2:

② $31612.991 \times 4 = 126451.964 \ m^3$

Section 3 - Method 1





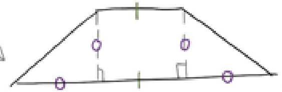


Part 3 Base View

- Section ①: Square Prism

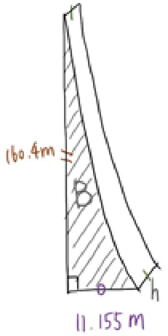
$$V_{①} = 18.65^2 \times 160.4 = 55790.729\ m^3$$

- Section ②



Section ② Base View

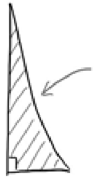
– Section ③



$$V_{③} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

shaded area

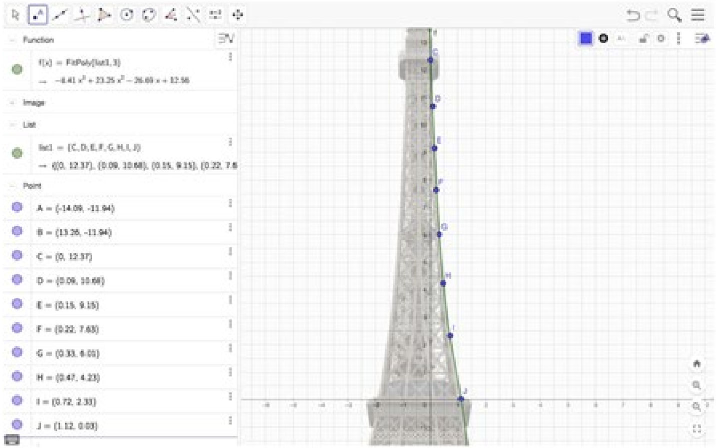
h



Can plot the points on the curve side to work out a fit function

Make the scale of 1 : 10 on the graph, which means that the length on the x-axis will be 1.1 units approximately.

Then the actual shaded area = area in the graph $\times 10^2$



Geogebra Modelling

$$\rightarrow \int_0^{1.12} -8.41x^3 + 23.25x^2 - 26.69x + 12.56 \, dx$$

$$\approx 4.90709$$

$$\rightarrow \text{Actual base area} = 4.90709 \times 10^2 = 490.709\ m^2$$

$$\rightarrow V_{③} = \frac{1}{2} \times 490.709 \times 18.65 = 4575.861\ m^3$$

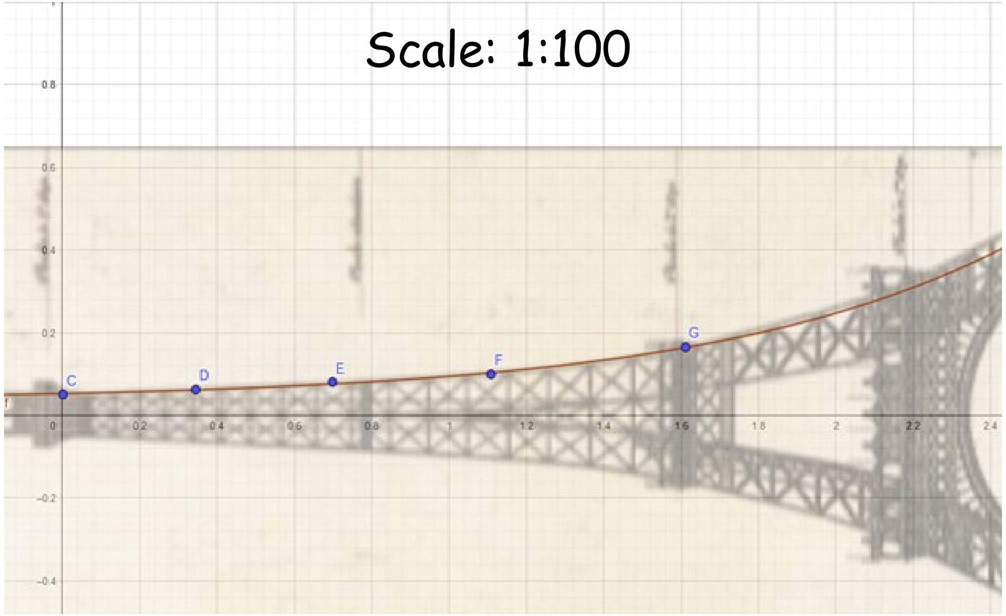
– Section ④ ⇒ * Simplify it to a triangular pyramid

$$V_{④} = \frac{1}{3} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{3} \times \left(\frac{1}{2} \times 11.155^2\right) \times 160.4 = 4989.804\ m^3$$

which means that the actual volume will be smaller

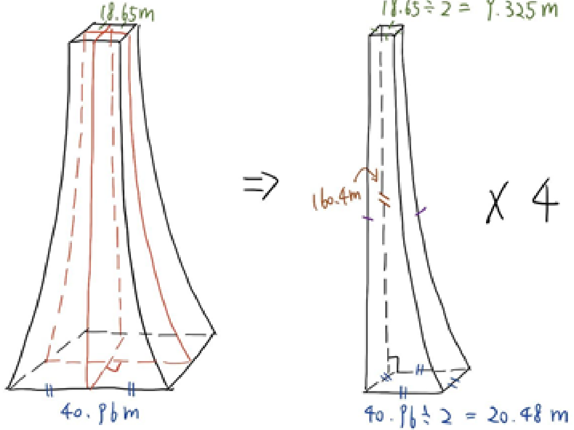
Section 3 - Method 2



$$\therefore \text{Total volume of Section ② } (V_②) = V_③ + 2 \times V_④$$
$$= 4575.861 + 2 \times 4989.804$$
$$= 14555.469 \text{ m}^3$$

$$\text{Total volume of Part 3 } (V_c) = V_① + 4 \times V_②$$
$$= 55790.729 + 4 \times 14555.469$$
$$= 114012.605 \text{ m}^3$$

METHOD 2



$$f(x) = -\frac{0.03}{1-1.48e^{-0.15x}}$$

length of ① $= -\frac{0.03}{1-1.48e^{-0.15a}}$

area of ② $=$ ①^2 $= y^2$

$$= \left(-\frac{0.03}{1-1.48e^{-0.15a}}\right)^2$$

$\therefore$ volume (of the small cuboid) $=$ area of ② $\times dx$

$= y^2 \times dx$

$\therefore$ Volume (of $\frac{1}{4}$ of whole part) $= \int_0^{1.604} y^2 \, dx$

$$= \int_0^{1.604} \left(-\frac{0.03}{1-1.48e^{-0.15a}}\right)^2 dx$$

≈ 0.019176959

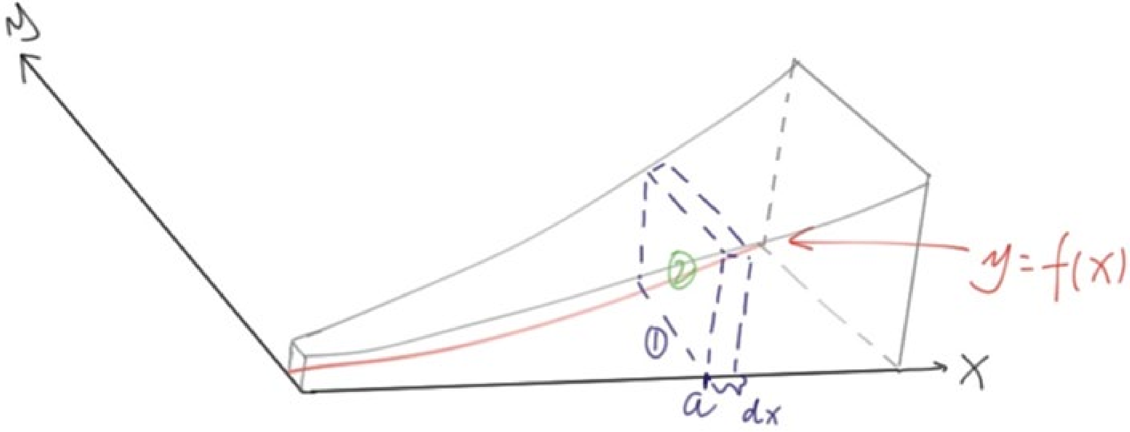
$\Downarrow$

1 : 1000000

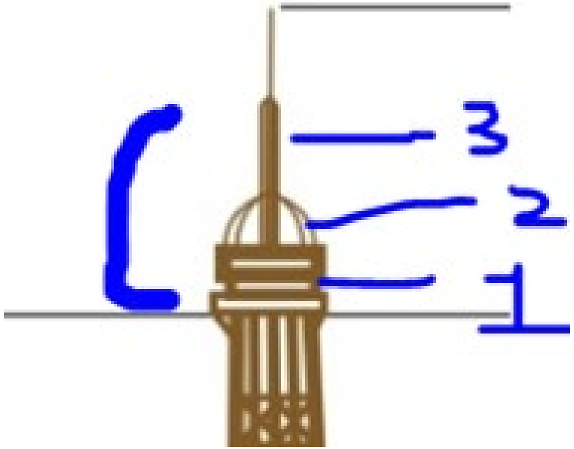
$\therefore$ Volume of $\frac{1}{4} = 19176.959 \, m^3$

Total volume $= 19176.959 \times 4$

$= 76707.836 \, m^3$

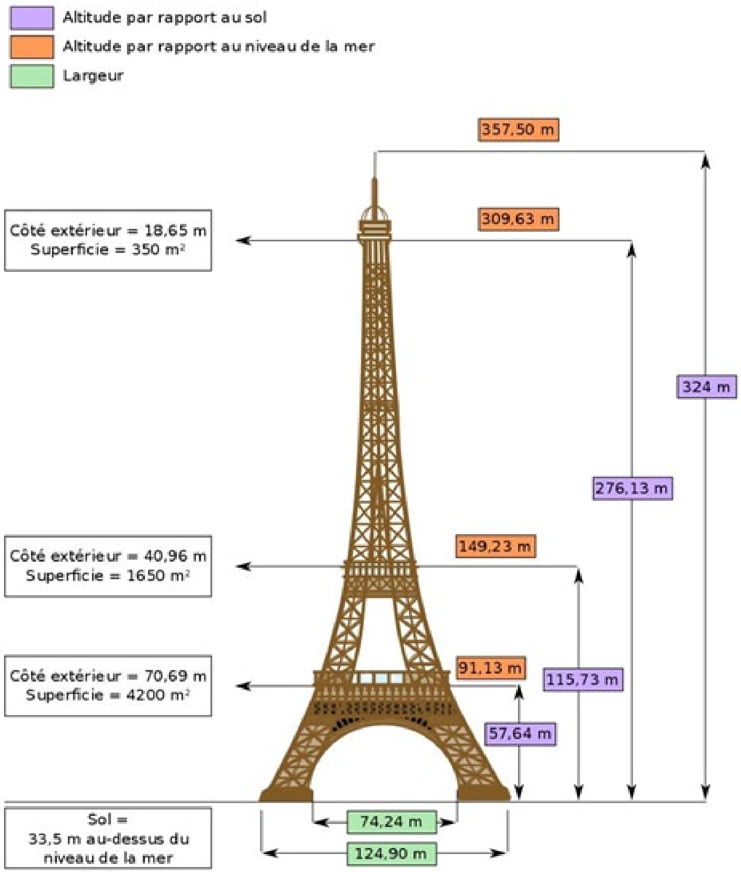


Section 4 (The top one)



The semi-circular shape on the 2D model actually represents something similar to a cube on the actual structure. Therefore, we decide to calculate the volume of two cubes and a cylinder. We think this is accurate enough and it's the best we can do.

sophie



I first used geogebra to figure out the scale: 10.34 cm :32400 cm = 1:3132.53cm

I then found all the length needed to calculate the volume of the two cubes and a cylinder.

<u>1(cube):</u>

Length = width= 1679.4746cm

Height=1017.1325cm

We assume all the cross sections of eiffel tower are squares. Therefore width equals to length.

Volume of 1 (cube) = 2868959460cm3

2(cube):

Length = width = 1253.70116cm

Height = 733.2626cm

Volume of 2 (cube) = 1152517656.4cm3

3(Cylinder):

Height = 2109.32cm

Radius = 147.683cm

Volume of 3 (cylinder) = 144528450.86cm3

4(cylinder):

Height = 1268.74cm

Radius = 50cm

Volume of 4 (cylinder) = 9964660.488cm3

TOTAL VOLUME OF THIS SECTION = 4175970227.7cm3
= 4175.970m3

Sara - available volume calculation

Layer 1: 565376.679 m^3

Layer 2: 126451.964 m^3

Layer 3: 76707.836 m^3

Layer 4: 4175.970 m^3

Total volume = 772712.449m^3

$$\text{volume of metal} = \frac{\text{mass}}{\text{density (iron)}}$$

$$= \frac{73\,000\,000 \text{ kg}}{7874 \text{ kg/m}^3}$$

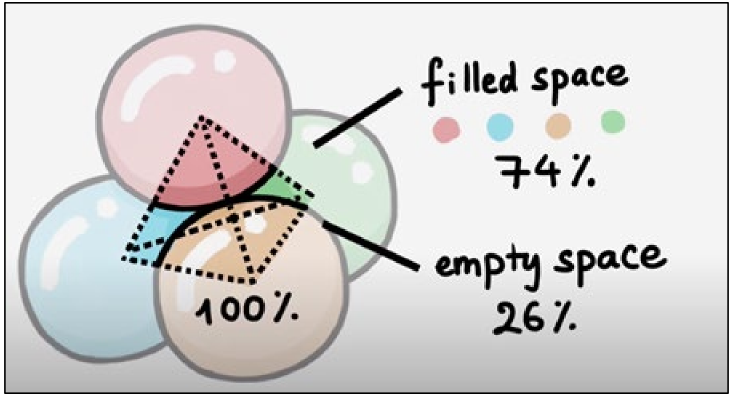
$$\boxed{= 9271.02 \text{ m}^3}$$

$$\text{available volume} = 772712.449 - 9271.02$$

$$\boxed{= 763441.429 \text{ m}^3}$$

final number of marbles

Average marble = 2cm^3
= 2x10^-6 m^3



space filled by marbles = available volume × 0.74

$= 763441.429 \times 0.74$

$\boxed{= 564946.658\ m^3}$

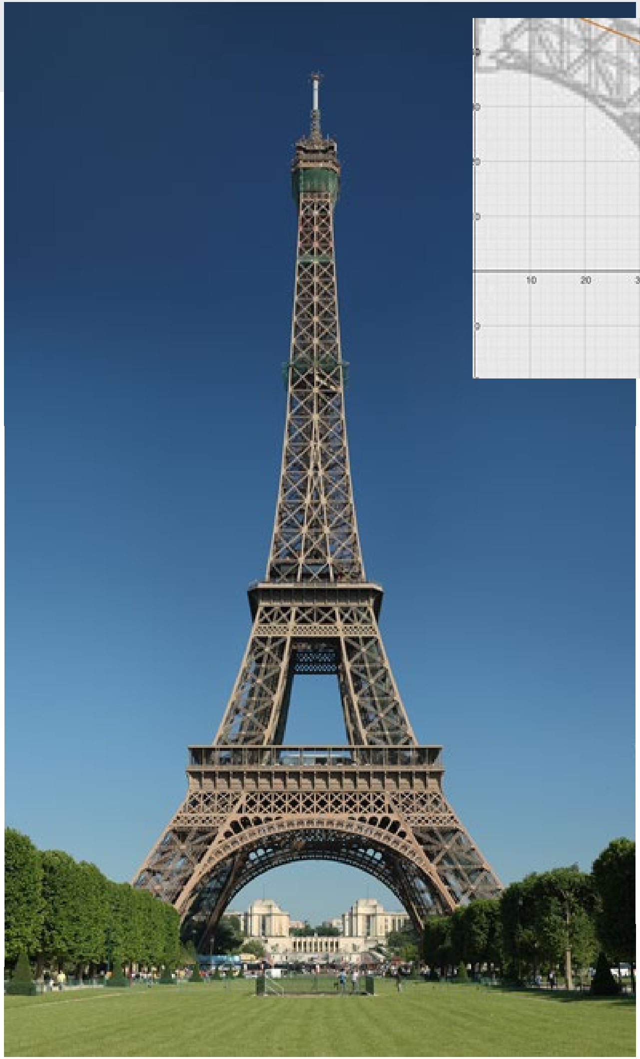
number of marbles $= \dfrac{564946.658}{2\times10^{-6}}$

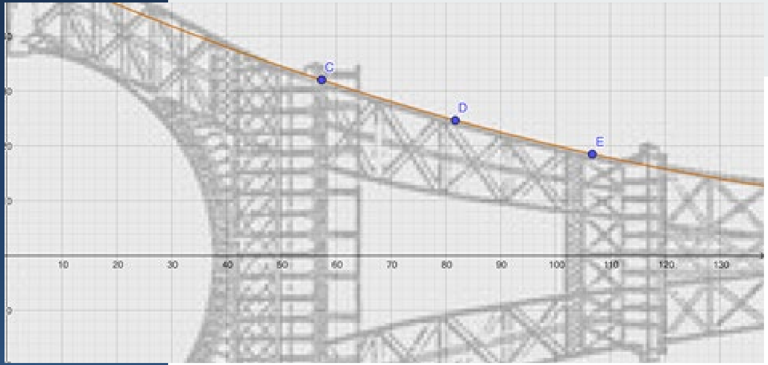
$\boxed{= 2.825 \times 10^{11}\ \text{marbles}}$

If 4 marbles are packed together as close as possible their centers form a tetrahedron. When there are more marbles a lattice is created, therefore the ratio of filled to empty space in the tetrahedron will be the same as the ratio for the available volume.

Evaluation

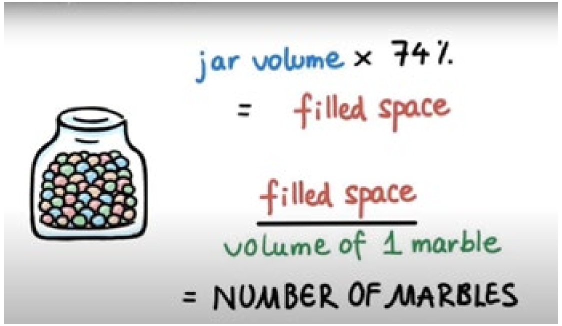
1. The shooting angle of the photo is an low angle, which largely affects the shape of the tower as well as the scale





2. There are bushes on the two bottom sides, parts of the tower are hidden

Evaluation



- When calculating the marbles, we considered the stacking efficiency, however we assumed the available space of the tower for marbles to be like the space in the jar. Yet, the inside space of the Eiffel Tower is divided into small sections by the metallic structure, that much space of the corners is not available to stack marbles, that the stacking efficiency in reality would be lower than expected.

Thank you!