Year 6

(Entry into Year 7)

15 Hour Revision

Mathematics

NORD
ANGLIA
EDUCATION

Section 1 – Numbers

4.5 hours

Integers, Powers and Roots (1.5 hours)

Integers

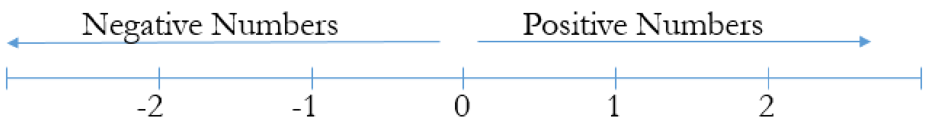
Much of the maths we encounter in everyday life is done with *integer* or whole numbers. These are numbers that are written without a fractional or decimal component. For example, the number 7 is an integer but the numbers 7.5 and $7\frac{1}{2}$ are not integers. Some quantities only make sense as integers. Can you describe what 2.96 automobiles or $1\frac{1}{2}$ children would look like?

1. Describe each number in the following list as an integer or non-integer.

5.4 9 7 -7.6 -2 $2\frac{1}{2}$ 1,000

Negative Numbers

Integers, like all numbers, may be positive or negative. Negative numbers lie to the left of zero on a number line:



We can describe the size of a number by its position on a number line. Since positive numbers are always to the right of zero on a number line, any positive number is greater in size than any negative number.

Determine which of the following numbers is greater in size.

2. -4, 0

3. 6, 2

4. -5, -6

5. 1000, -1001

6. 4, -5

Negative Numbers

Performing arithmetic with negative numbers requires a few rules. First, addition of negative numbers can be rewritten in the following manner:

$$-4 + 5 = 5 - 4$$

Adding a negative number is the same as subtracting that number after removing the negative sign. Here's some more examples of this in practice:

$$-2 + \ -2 = -4$$

$$-4 + \ 4 = 0$$

Multiplication and division work a little bit differently. To perform these operations, remember the following:

- If the two factors in multiplication have different signs then the answer will be negative.
- If the quotient and dividend in division have different signs, the answer will be negative
- For all other multiplications and divisions with two numbers, the answer will be positive.

See the examples below.

$$6 \times -6 = -36$$

$$-2 \times -10 = 20$$

$$18 \div -3 = -6$$

$$-8 \div -2 = 4$$

Calculate the answers to the following problems:

7. -23×4

8. $8 + \ -12$

9. $16 \div -8$

10. $-10 + 10$

11. $0 \div -2$

12. $12 \div -48$

13. -8^2

Factors and Prime Numbers

We have used the word *factor* to describe numbers that can be multiplied together to give us a product. Any integer will have at least two possible integer factors: itself and the number 1. If these are the only possible factors—that is, there is no way to write the number as a product of other sets of factors—then that number is *prime*.

The number 23 is a prime number because it has no integer factors besides 1 and itself. In the case below, the number 14 is not prime because there are possible factors besides 1 and 14

$$1 \times 14 = 14$$

$$2 \times 7 = 14$$

14 has factors of 1, 2, 7, and 14. Because it has factors besides 1 and itself, 14 is a *composite* number because it is *composed* of various factors. Some composite numbers can have many more factors. The number 24 has several:

$$1 \times 24 = 24$$

$$2 \times 12 = 24$$

$$3 \times 8 = 24$$

$$4 \times 6 = 24$$

In this case, 24 has factors of 1, 2, 3, 4, 6, 8, 12, 24. We normally write out a number's list of factors from smallest to biggest.

Find all the factors of the following numbers:

14. 121

15. 100

16. 21

17. 35

18. 72

19. 19

20. 1

Determine whether or not the following numbers are prime:

21. 5

22. 1

23. 2

24. 4

25. 29

26. 37

27. 53

A powerful method for finding all the prime numbers in a given range is the *Sieve of Eratosthenes*. This can be done via writing out the list of all numbers which you would like to check. Then, select a factor and cross out all numbers which include that factor, since this would make them non-prime. The diagram below depicts how to do this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15

We begin by creating a grid with all the numbers to check whether or not they are prime, or their *primality*.

1	~~2~~	3	~~4~~	5
~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15

The next step is to cross off every multiple of 2.

1	~~2~~	~~3~~	~~4~~	5
~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	15

Keeping track of which numbers are already marked off, we continue on by crossing off every multiple of 3, and repeating this with 4 and 5.

1	~~2~~	~~3~~	~~4~~	5
~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~

Ultimately, we are left with a list of numbers which do not have any of these factors. The numbers 5, 7 and 11 are all prime. The number one also remains on this grid but it is a special case. The number 1 is never prime!

28. Use the Sieve of Eratosthenes to find the prime numbers between 1 and 25 using the grid shown below:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Multiples

When two numbers are both factors of another, larger number then they share a common *multiple*. Of special interest is the *lowest common multiple* of two factors which is the smallest number which both factors divide into evenly. Let's illustrate this by finding the lowest common multiple of 3 and 4.

Here's a list of the multiples of 4 up to 40, in order:

4 8 12 16 20 24 28 32 36 40

The following is a list of multiples of 3 up to 39:

3 6 9 12 15 18 21 24 27 30 33 36 39

To find common multiples of 3 and 4, we check to see which numbers show up on both lists. The common multiples are 12, 24 and 36. These can all be divided evenly by the factors of 3 and 4. The lowest common multiple is thus 12 as it is the smallest.

Use this method to find the lowest common multiple of the following number pairs:

29. 5 and 3

30. 2 and 7

31. 6 and 5

32. 4 and 5

33. 2 and 3

34. 10 and 5

35. 6 and 8

Square and Cube Roots

The square of a number is the number multiplied by itself. The cube root is the number multiplied by itself twice.

$$2^3 = 2 \times 2 \times 2$$

$$4^2 = 4 \times 4$$

For the above examples, we would say that the number 4 is the square root of 16 and 2 is the cube root of 8.

Find the square or cube of the following numbers:

36. 3^3

37. 12^2

38. 10^2

39. 4^3

40. 5^3

41. 10^3

42. 4^2

Find the square root of the following numbers:

43. 144

44. 169

45. 81

46. 121

47. 196

Fractions, Ratios and Rounding (1.5 hours)

Fractions

Fractions, decimals and percentages can all be used to express quantities in between whole numbers.

$$\frac{1}{2} = 0.5 = 50\%$$

$$1\frac{3}{5} = 1.6 = 160\%$$

$$\frac{9}{1} = 9.0 = 900\%$$

In a fraction, if the numerator is greater than the denominator then it is an *improper* fraction. The fraction $\frac{9}{1}$ is an example of this.

To convert a fraction into a decimal, divide the numerator by the denominator. Here's an example.

$$\frac{9}{2} = 9 \div 2 = 4.5$$

To convert a decimal into a percentage, multiply the decimal by 100.

$$0.9 \times 100 = 90$$

$$0.9 = 90\%$$

Converting a fraction into a percentage requires both steps listed above.

Convert the following decimals to fractions or improper fractions. One method of doing this is to guess-and-check dividing various numbers with a calculator.

1. 9.2

2. 1.75

3. 2.5

4. 0.40

5. 0.875

6. 0.35

7. 0.99

Convert the following fractions or improper fractions to decimals:

8. $\frac{6}{5}$

9. $\frac{3}{4}$

10. $\frac{11}{10}$

11. $\frac{8}{16}$

12. $\frac{5}{15}$

13. $\frac{12}{100}$

14. $\frac{7}{10}$

15. $\frac{8}{50}$

Determine which of the following is greater:

16. 19% or 0.85

17. 0.22 or one-fifth

18. 80% or $\frac{1}{2}$

19. $\frac{2}{3}$ or 70%

20. 2 or 130%

21. 1.99 or 200%

22. 60 or $\frac{6}{10}$

23. 150% or 1.60

Answer these word problems:

24. Children make up approximately fifteen percent of the population of Dubai. If there are 2,000,000 people in Dubai, how many of them are children?

25. Carol wants to give two thirds of her cookies to her younger brother Daniel. If she has 9 cookies, how many will she give Daniel?

26. There are 100 honeybees in a colony. After a cold winter, 22 percent of them have flown away. What is the percentage of honeybees that remain?
    a. Express this percentage as a decimal

27. There are 180 degrees in the sum of all angles in a triangle. What fraction of this number is 60 degrees?

28. A petrol station is selling fuel for two dollars a litre. Another shop nearby is selling the same fuel for \$2.40 per litre, but is offering a discount on this price of 20%. Which is a better deal, the first or the second?

Fraction Arithmetic

To multiply a fraction by an integer, the product is the numerator times the integer number:

$$\frac{1}{2} \times 3 = \frac{3}{2}$$

$$\frac{5}{2} \times 2 = \frac{10}{2}$$

Find these products:

29. $\frac{3}{4} \times 2$

30. $\frac{9}{10} \times 10$

31. $\frac{5}{6} \times 3$

32. $\frac{5}{8} \times 1$

33. $22 \times \frac{1}{11}$

Multiplication with two fractions is done by multiplying the numerators together to get the numerator of the product, and then multiplying the denominators of the factors to get the denominator of the product. Here's an example:

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

Numerator: $1 \times 3 = 3$

Denominator: $2 \times 4 = 8$

34. Perform these fraction multiplications:

a. $\frac{2}{3} \times \frac{5}{6}$

b. $\frac{4}{2} \times \frac{2}{2}$

c. $\frac{1}{10} \times \frac{1}{10}$

d. $\frac{7}{8} \times \frac{1}{7}$

e. $\frac{10}{2} \times \frac{3}{2}$

To perform addition and subtraction with fractions, we need to remember two rules.

- Fractions can only be added if they have the same denominator (the bottom number in the fraction)
- To add two fractions together, add the top numbers and place this over the shared denominator.

In practice, adding fractions looks like this:

$$\frac{4}{9} + \frac{1}{9} = \frac{5}{9}$$

$$\frac{10}{10} + \frac{1}{10} = \frac{11}{10}$$

$$\frac{17}{18} - \frac{5}{18} = \frac{12}{18}$$

What if two numbers do not share the same denominator? The only way we can change the denominator is by multiplying it. To do this without changing the value of the number, we multiply by a fraction which is equal to one:

The sum shown below cannot be added in its current form because the denominators are different.

$$\frac{5}{8} + \frac{1}{4} = ?$$

Our first step in solving this is to rewrite $\frac{1}{4}$ into an equal fraction with 8 in the denominator. The way to do this is to multiply it by $\frac{2}{2}$.

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$$

We've chosen $\frac{2}{2}$ because it is equal to 1. Multiplying any number by 1 leaves it unchanged. We can then write our original sum like so:

$$\frac{5}{8} + \frac{2}{8} = ?$$

Then, we just need to add the top numbers and leave the bottom number unchanged:

$$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$$

Subtraction works the same way. Make sure both fractions have the same denominator, then subtract the top numbers.

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

Find these sums or differences:

35. $\frac{7}{8} - \frac{1}{4}$

36. $\frac{11}{12} + \frac{1}{12}$

37. $\frac{3}{4} + \frac{1}{8}$

38. $\frac{5}{4} + \frac{3}{16}$

39. $\frac{9}{10} + \frac{2}{5}$

40. $\frac{1}{2} + \frac{1}{2} + \frac{1}{4}$

41. $\frac{1}{3} + \frac{2}{9}$

Ratios (1 hour)

When we want to compare two numbers, we are often interested in how the first corresponds to the second. For example, if we want to find the number of red cars in a parking lot with many different colours of cars, we would express this as a *ratio*. This is written in the form of a fraction:

$$\frac{number\ of\ red\ cars}{total\ number\ of\ cars}$$

If there are 8 red cars in a parking lot containing 90 cars, the ratio is $\frac{8}{90}$. To state this in words, we say that there are *8 red cars for every 90 cars* or *8 to 90*. To put it simply, a ratio is just a comparison between two different numbers.

1. A scientist performed a study of the numbers of birds and mice in a park. She found that the ratio of mice to birds is $\frac{3}{1}$.
   a. If there are 10 birds total in the park, then how many mice are there?

2. James counts seven cherry candies for every three strawberry candies in his bag. Write this as a ratio of cherry to strawberry candies.

3. If there are 2 widgets for every gizmo, and 2 gizmos for every thingy, then what is the ratio of widget to thingy?

Proportion

If two quantities have the same ratio, then they are *proportional*. This means they have the same relative size. The quantities $\frac{8}{4}$ and $\frac{4}{2}$ are proportional because the quotient of 8÷4 and 4÷2 is 2.

4. Are the quantities $\frac{10}{5}$ and $\frac{20}{10}$ proportional?

5. Are the quantities $\frac{2}{3}$ and $\frac{1}{4}$ proportional?

Simplifying Fractions

To simplify a fraction, find a number which divides both the numerator and denominator. For example, to simplify the improper fraction $\frac{30}{5}$ we would divide both the top and the bottom by 5 to get $\frac{6}{1}$. The fraction $\frac{24}{64}$ can be simplified into $\frac{3}{8}$ by recognizing that both 24 and 64 are divisible by 8. If it is difficult to figure out which number to divide by, start with a small one. Let's use the example of $\frac{144}{48}$. It's not very easy to see which factors 144 and 48 share in common. We'll do this by dividing both numerator and denominator by 2 or 3 as many times as we can:

$$\frac{144}{48} \to \frac{72}{24} \to \frac{36}{12} \to \frac{18}{6} \to \frac{9}{3} \to 3$$

In this instance, we divided both top and bottom by 2 or 3 several times and ended up with the answer of 3. A quick check with a calculator confirms that this is indeed the case.

Using this method, simplify the following fractions if possible. In some of these you may need to divide by 4, 5, 6, or even 7.

6. $\frac{96}{12}$

7. $\frac{100}{30}$

8. $\frac{18}{36}$

9. $\frac{15}{25}$

10. $\frac{2}{4}$

11. $\frac{21}{35}$

12. $\frac{17}{21}$

13. $\frac{12}{18}$

14. $\frac{19}{57}$

Calculations (30 minutes)

Decimals

Perform the following calculations with decimals:

1. $12 - 14.552$
2. $0.581 + 1.662$
3. $100 - 0.911$
4. $2.212 + 1.121$
5. $1953 - 7.62$

6. $8.639 + 1.998$

7. 6.8×2

8. 20×1.5

9. 1.99×2

10. 0.87×7

11. $1.9 \div 2$

12. $4 \div 0.7$

13. $12 \div 0.60$

14. $0.08 \div 20$

15. 19×0.19

Evaluating Calculations with Several Numbers

When performing operations on multiple numbers at once, we can change the order in which they are done with brackets. Let's say we want to find a sum of products:

$$4 \times 2 + 3 \times 5 + 2 \times 1 = 25$$

In this case, we evaluate each product first then add them together because multiplication and division occur before addition and subtraction. We can change this order with brackets, however:

$$4 \times (2 + 3) \times (5 + 2) = 140$$

Here, we evaluate whatever is in the brackets and then perform the multiplication. The order of operations which determines this is the acronym *BODMAS* which stands for:

Brackets

Order & Indices - also known as exponents (such as the 3 in 4^3).

Division & Multiplication

Addition & Subtraction

To use this rule, we first evaluate everything in brackets, followed by the indices or exponents, followed by division/multiplication and addition/subtraction. If done properly, there should be no difference between doing multiplication before division and division before multiplication. The same rule applies to addition and subtraction.

Let's use these rules to do another example:

$$2 + 3 \times (4 + 5) \div (2 - 1) = ?$$

Our first job is to figure out what is inside the brackets. The numbers above can be written as:

$$2 + 3 \times (9) \div (1) = ?$$

Then, we perform multiplication and division:

$$2 + 27 =?$$

And finally, addition / subtraction:

$$2 + 27 = 29$$

In the following questions, use the BODMAS method to properly calculate the result.

16. $3 + 2 \times (3 + 2)$

17. $5 + 5 + 5 \times 2$

18. $7 + 1 \times (2 - 3) \div 2$

19. $4^2 + 3 - 9$

20. $2 \times 9^2 \div 3$

Section 2 – Algebra

3.5 hours

Expressions and Formulae (2.5 hours)

Expressions and Terms

When we use letters or other variables in algebra, it's helpful to be able to describe them alongside numbers. In an equation, a single number, variable or product of the two is called a *term*. Let's look at an example. In the equation below, there are four terms:

$$\frac{2a}{10} + 3b + 4 = 10$$

There terms are $2a, 3b, 4$ and 10. We use this word because it makes it easier to describe what we are doing with algebra. It would be very clumsy to say *add the product of 2 and a which is divided by 10* every time. Instead, we could say *add the first term* because $\frac{2a}{10}$ comes first in the equation above.

An *expression* is a collection of terms. $3b + 4$ is an expression. $\frac{2a}{10} + 3b + 4$ is also an expression. 10 by itself is not an expression because it does not include any kind of operation (addition, multiplication and so on).

1. In the equation $15x + 2 - 3 = 5$,
    a. What is the first term?

    b. What is the second term?

    c. What is *x*?

Don't worry if you cannot answer part C above. We will come back to solving equations in a bit.

Algebra and Equations

Why do we use algebra? Not because it's always easy and fun—though it definitely can be!—but rather because it is the only way to describe a huge range of problems that we encounter in daily life. Often, we are doing algebra without even realizing it. Whenever we want to find an unknown quantity from information we already know, we are going to eventually use algebra. We can illustrate this with an example.

If you are getting paid five dollars a day to deliver newspapers in the neighbourhood and you already have ten dollars, how long will it be before you can afford a 50 dollar skateboard?

There are several steps in setting up the algebra problem and solving it. First, we have to identify what we know and what we still need to find out. In this case, we want to find the time it will take before we can buy the skateboard. We know how much money we have as well as how much additional money we will get per day. Let's use the letter *d* for days to represent how long it will take before we can afford the skateboard.

We know that we need to save fifty dollars. So, one side of our equation will be fifty:

$$? = 50$$

What is our total amount of money? Here, it's ten dollars plus five dollars for every day that you deliver papers. The amount of money you have is $10 + 5 \times d$. Since we want to know the number of days it takes for our money to equal the price of the skateboard, we set this expression equal to 50:

$$10 + 5 \times d = 50$$

To solve this equation, we would need to isolate *d*. That is, we want to get it by itself on one side. When we work with equations like these, there are two rules:

- Use operations to cancel out numbers and eliminate them from one side
- Any operation done must be done to *both* sides otherwise the equation is no longer true.

To find *d* above, we would subtract 10 from 50 to get 40 and then divide 40 by 5. Then,

$$d = 8$$

It would take 8 days to save enough money for the skateboard.

Algebra equations are ways of stating facts. We could say that $2 = 2$. But, if we multiply just *one* side of the equation by 3, then it looks like this: $3 \times 2 = 2$ which is very, very wrong. The numbers 6 and 2 are not the same! This is what happens if we forget to perform our operations on both sides of the equation.

For the following problems, write out an equation with variables and numbers and use it to answer the question.

2. The number of chickens in a barn is seventeen multiplied by two. How many chickens are there?

3. Yesterday, there were nine candy bars left in your backpack. Today, there are three. How many candy bars disappeared?

    a. It turns out that a group of people walked by and each person took exactly two candy bars. How many people were in the group?

4. After getting his pay check, Yousef must pay fifty percent of his income in taxes and *then* another twenty dirhams for rent. If his income is 100 dirhams, how much money does he have left over?

5. The temperature is rising by 5 degrees every hour. If it is 22 degrees now, how long will it be before it is 45 degrees?

6. What is the formula for changing the number of hours into an equivalent number of minutes? Use the letter *h* for hours and the letter *m* for minutes.

Rounding and Estimation

Sometimes, we are only interested in a *rough* quantity, or one that is not exact. If we want to buy 21 candy bars at 3.98 dirhams apiece, then we are can approximate by replacing the numbers given with ones that are close to the original but easier to multiply. The multiplication 4×20 is much easier than calculating 3.98×21. Our answer won't be exactly correct, but it will give us a quick estimate.

Use estimation to solve the following:

7. 11 Merchants each brought 31.2 kilograms of goods to the bazaar. Roughly how many kilos did they bring in total?

8. Hassan is paid 29.55 dirhams an hour and usually works for 7.109 hours each day. How much is he paid each day, on average?

9. 101.263×99.2

10. $501.8 \div 47$

11. $5.9844 - 3.102$

Like Terms

When an expression has all numbers, like this one: $6 - 2 + 1 \times 4 + 3$, we can perform any multiplications/divisions, add up all the numbers and we should be left with just one term. If there are variables, this does not work. We can't add together $3a + 2b$ into just one term because we don't know what they are yet! We can, however, add all like terms. Two terms are like if they have the same variable or if they are both numbers. $2a$ and $6a$ are like terms and they add up to $8a$. 6 and $2a$ are not. $2b$ and $2a$ are not like terms either.

Simplify the following expressions by combining like terms:

12. $4a - 7b + 3a$

13. $2x - 3 + 3y - 4x$

14. $h - 9h - 2k$

15. $\frac{h}{2} + \frac{h}{4}$

16. $d + d + d + d + d + d + c + c + a$

17. $x \div \sqrt{16}$

Algebraic Operations

If a number is right next to a bracket without an operation, we assume that it is multiplying the bracketed terms. Thus, $2(3 + 1) = 8$ and $3(a + b) = 3a + 3b$.

Calculate or simplify the following expressions:

18. $2(9 - 5)$

19. $\frac{a}{2}(7 + 3)$

20. $\sqrt{x^2}$

21. $2x + 3x + 3 + 5$

22. $2c \times 2c$

23. $(8b)(2)$

24. $(5 + 2)^2$

25. $(\sqrt{5+4}\ \times 3b)$

Solving Equations

Use the rules from the *Algebra and Equation* section to solve for the unknown quantity in the following:

26. $9b + 1 = 10$

27. $\frac{x}{2} = 5 - 3$

28. $9 - 2a = 5$

29. $x + x + x + 3 = 27$

30. $\frac{13}{4}y = 26$

Sequences

A sequence is a bunch of objects which follows a pattern. We're most interested in sequences of numbers and variables. You might have already come across sum in the earlier problems in this book. Sequences all have *patterns* which describe what numbers are in the sequence. Here's a sequence with the pattern *add four to the previous number to get the next number*.

4 8 12 16 20 24 28 32 36 40

Here's one with the pattern *multiply the previous number by 2:*

2 4 8 16 32 64 128 256 512 1024

This one is more complicated. In this pattern, we *add two then multiply by 3.*

2 12 42 132

In all of these sequences, the numbers would go on to infinity. We have written down just a few of them. In each case, we must also have an initial number to start with. In the first sequence, it was four and in the second it was 2. We have chosen these and they could have been anything.

31. Find the terms which are missing in the sequences shown below:
    a. 1 ___ 5 7 9 11 13

    b. 128 64 32 16 __ 4 2 1 $\frac{1}{2}$

    c. 15 12 __ 6 3 0 -3

    d. 5 25 125 __ 3125 15625

e. 1 ___ 3 4 5 ___ 7 ___ 9

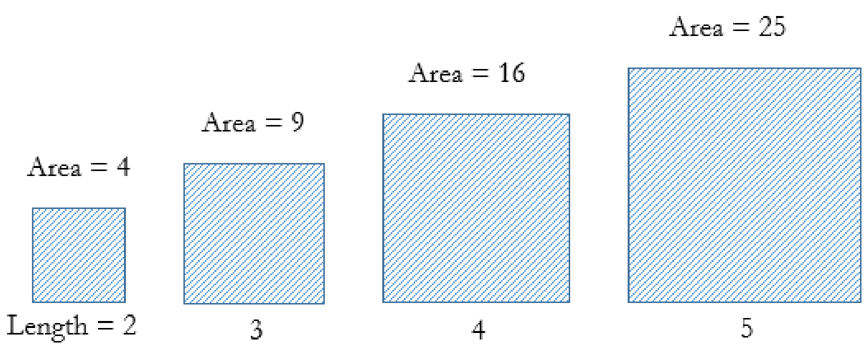
f. $2x$ ___ $6x$ $8x$ $10x$

If we can describe a sequence with just addition and subtraction, it's an *arithmetic* sequence. Exercises *a, c, e* and *f* in problem 31 are all arithmetic sequences. The difference between each term should be a constant amount. There are other types of sequences, though. Exercises *b* and *d* describe *geometric* sequences. To find a number in these types of sequences, we divide or multiply the preceding number.

Why do we use the word geometric? It's because they're related to the geometry of shapes!

Here's an illustration:

Sequence: 4 9 16 25



For these squares, we find the area by multiplying side lengths. In geometric series, we multiply by a fixed number.

32. Describe the following sequences as arithmetic or geometric:

a. 2 4 6 8 10

b. 3 6 12 15 18

c. 7 14 21 28 35

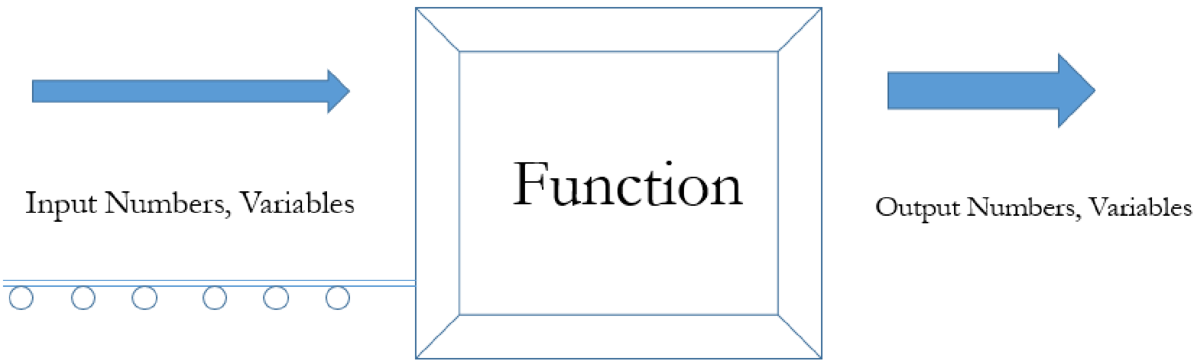
d. 7 14 28 56 102

e. -1 2 -4 8 -16

Functions (1 hour)

In mathematics, we spend quite a bit of time talking about numbers but we care just as much—if not more—about how those numbers can change and transform in various ways. We've seen how multiplication and addition can make numbers larger while subtraction and division make them smaller. Functions are a way to keep track of how all numbers change through various operations, as opposed to just a single calculation. A function tracks how any number might look after being doubled or added to.

It's helpful to think of functions as a machine which takes an input quantity and spits out an output. There is one and only one output for every input. There can be multiple inputs per output, however.



Functions are described in terms of their variables. A function of the variable x would be written as $f(x)$. We write out functions like this:

This tells us to take any number fed into the function, double it and add 3.

$$f(x) = 2x + 3$$

The only variable which appears in this equation is x so this is a function of x.

If we used an x value of 8, the value of $f(x)$ would be 19 because $f(x) = 2(8) + 3$.

Answer the following questions:

1. Use the value of $a = 12$ in the function $f(a) = \frac{a}{4} + 3$
   a. What's the output?

   b. What if the value of *a* is sixteen instead?

   c. This function will always output or *return* a positive number if you plug in a positive number. What will happen if you plug in a negative number? Will it return positive? Negative? A mix?

2. Write a function which takes in a variable y and returns a value of three times y.

3. Thomas has found a strange mathematical object which takes in an input and returns both -3 and 3 as valid outputs. Is this a function? Why or why not?

4. On a farm, two new calves are born every year and farmer Joe buys one more from the market each year. Write a function that takes in the number of cows he has from the previous year and returns the number of cows he has in the current year.

5. The function below takes in the length of the sides of a square and returns the area of that square.

$$f(a) = a^2$$

Write out a function that takes in the side lengths of a cube and returns the cube's volume.

Graphs (1 hour)

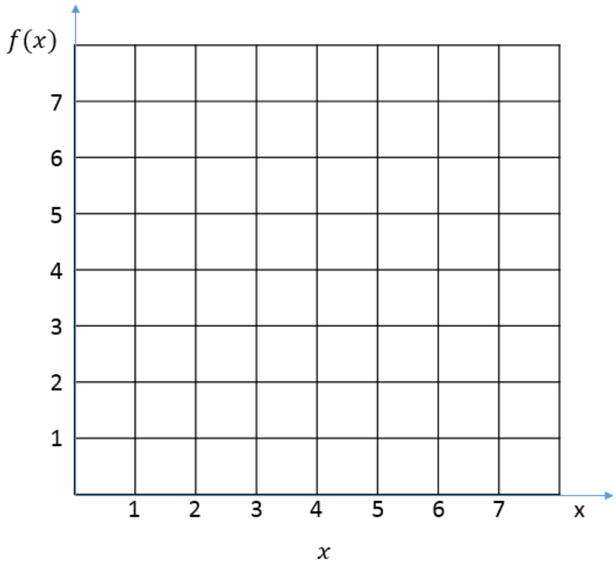
If we want to see how a function behaves with a lot of numbers, we'll have to make a table to compare the inputs with an output. Let's use our function from before, $f(x) = 2x + 3$. The table below has the input on the left and the output on the right.

$\boldsymbol{x}$	$\boldsymbol{f(x)}$
0	3
1	5
2	7
3	9
4	11
5	13
6	15

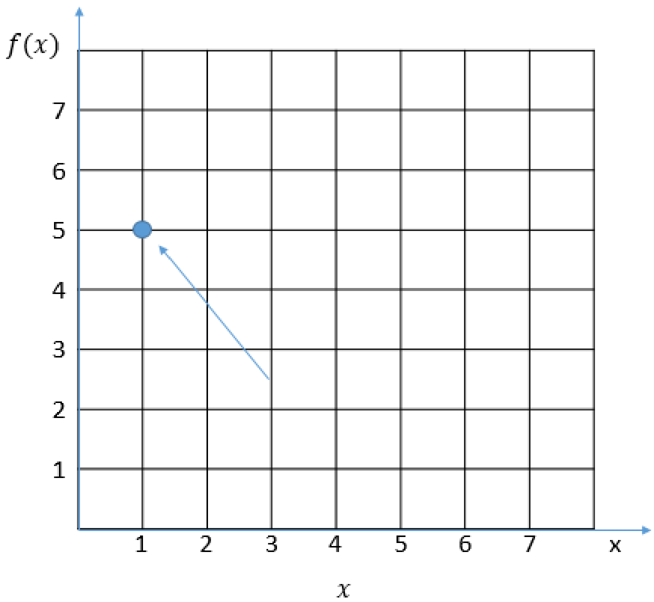
1. What is the value of this function if $x = 6$?

This table works fine for a few points. But, if we want to know how more about how this function *behaves* or how it transforms numbers, we need something more visual. We'll need to use a graph.

The most common type of graph is a Cartesian plot. You've undoubtedly seen these before. The plot below has two axes, one each for *x* and *f(x)*

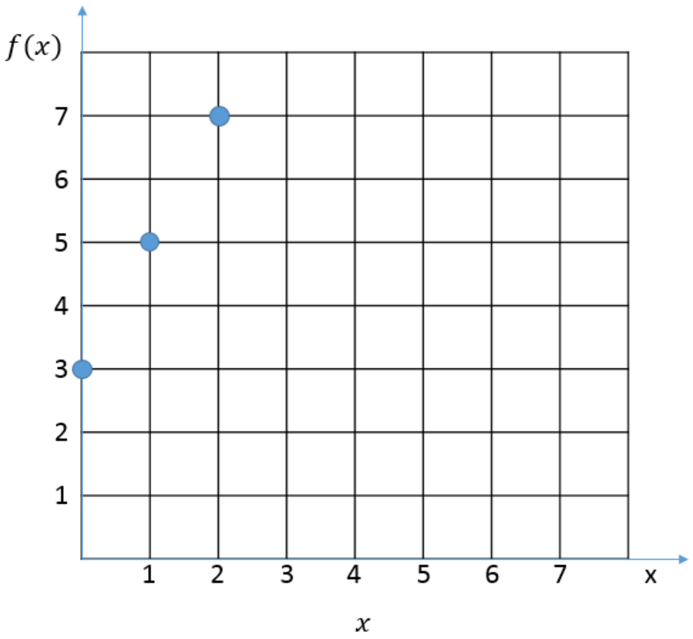


To use this plot, we place pick pairs of numbers from our table above and put them on the plot. Let's start with the point $= 1\ , f(x) = 5$:

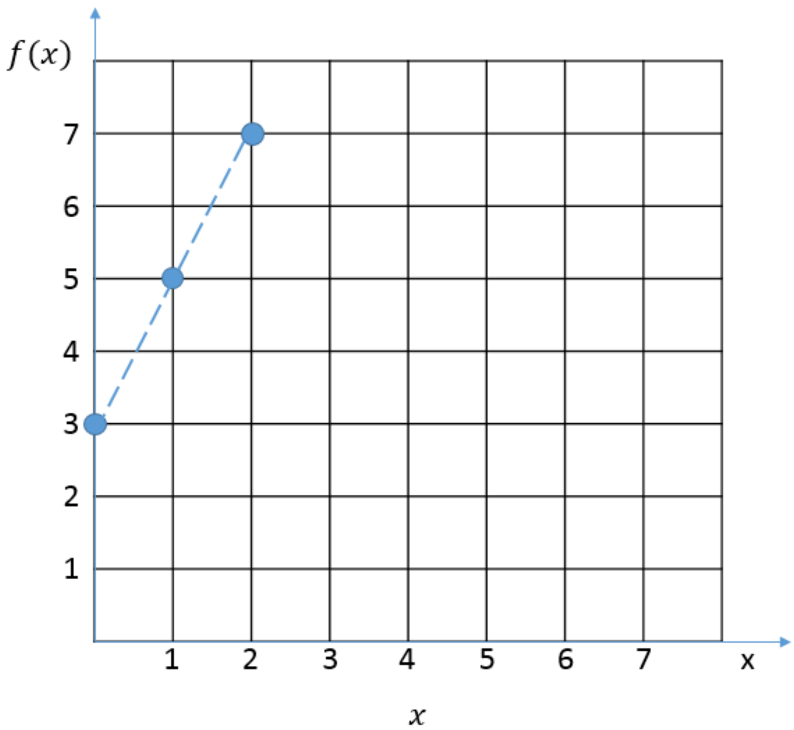


The point which placed sits on a line corresponding to $f(x) = 5$ and $x = 1$. These two lines intersect and that is where we should place out point.

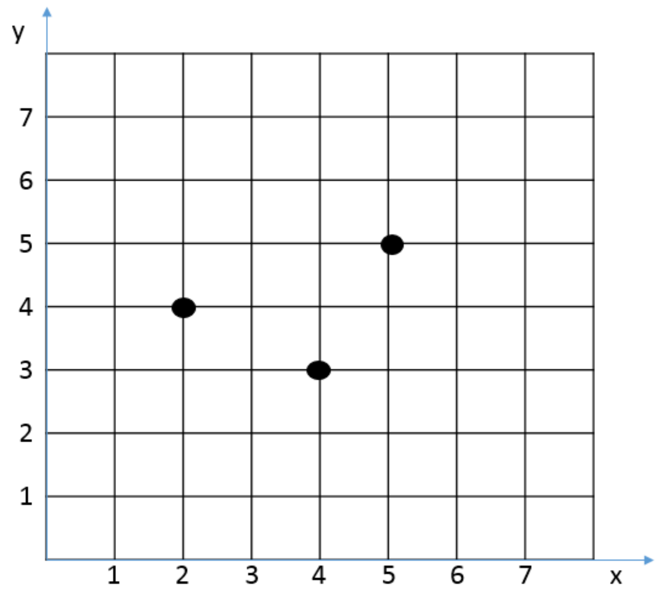
If we place a few more points from our table, the plot looks like this:



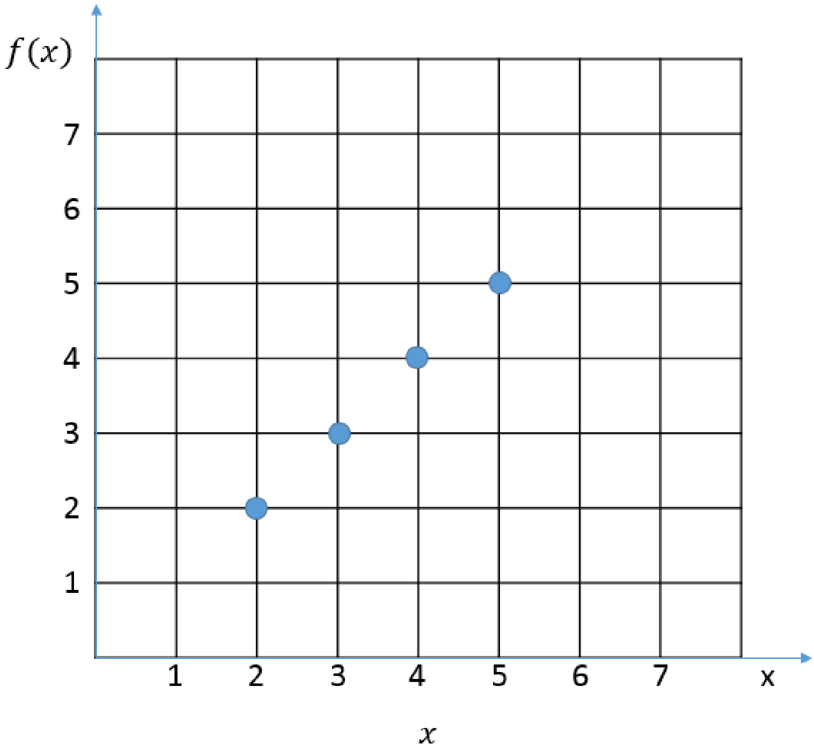
We can connect these points to draw a line that represents our function:



Not all functions are going to have points that sit on lines. The plot below, for example could be a function but there's no way to draw one straight line through all the points.



For the following problems, use the plot shown below:



2. The four points on the plot appear to be on a line. Draw in two more points that might be on this line.

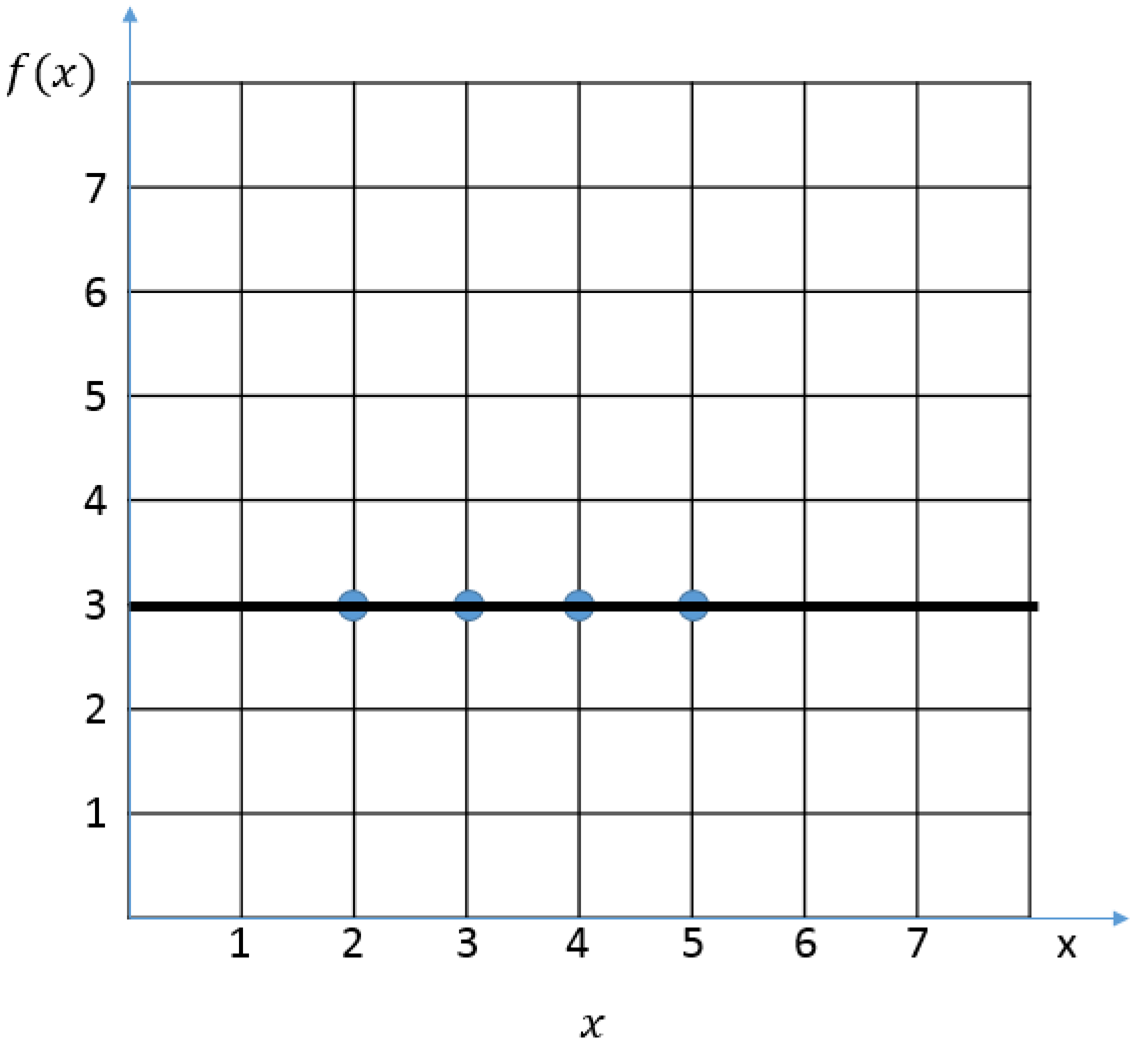
3. Fill out this table which compares *x* with *f(x)*.

x	$f(x)$

4. For every increase of 1 in *x*, *f(x)* also increases by 1. This means the two quantities are *proportional*. Write out a function *f(a)* that has an output which increases at the same rate as its input.

Parallel Graphs

Let's examine the graph below.



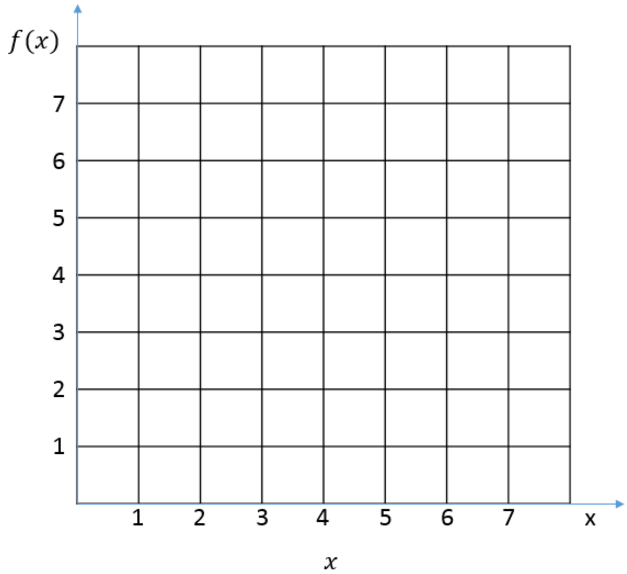
This graph has four points which all lie on the same horizontal line. It doesn't matter what the value of x is because every output is $f(x) = 3$. This is still a function (because there is one and only one possible output) but it's a little boring.

The function's line is *parallel* to the x-axis. That is, it goes in the same direction as the axis and will never cross the x-axis. To represent this in an equation, we write this out as:

$$f(x) = 3$$

Note that x doesn't show up at all! Again, because it doesn't matter what x is we are free to ignore it in our equation.

5. On the blank graph below, draw lines for the functions $f(x) = 2$, $f(x) = 1$ and $f(x) = 4$.



6. Are these functions all parallel? Why or why not?

7. Sketch out the graph of the function $f(x) = x^2$ on the plot above. Note that this will *not* be a straight line. You will need to draw an *arc* or curved line to connect all the points

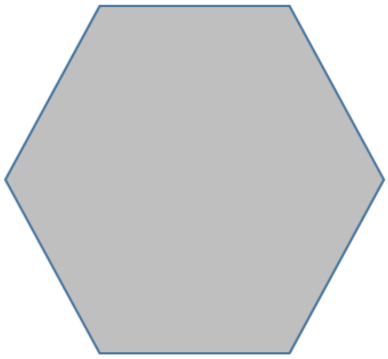
8. Sketch out the graph of the function $f(x) = \frac{x}{2}$ on the plot above.

Section 3 – Geometry

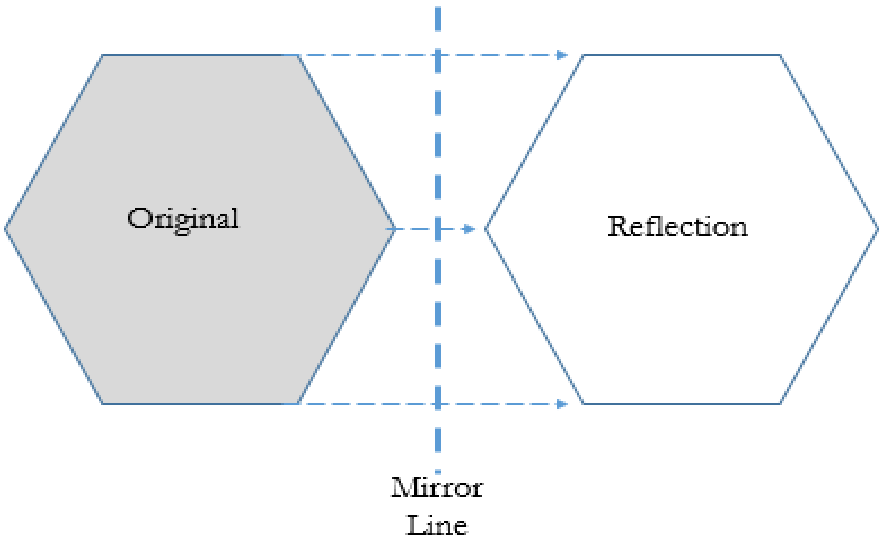
2.5 hours

Symmetry (1 hour)

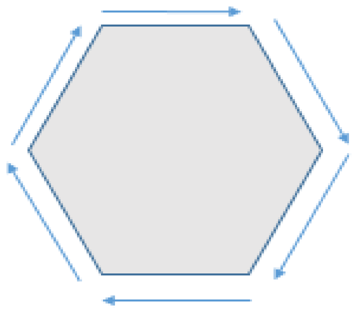
Symmetry is the measure of how much objects repeat themselves. Humans are symmetric because our left side looks the same as our right side (relatively speaking). Squares are even more symmetric because they can be flipped upside down or right to left and they will still look the same. Circles have *rotational* symmetry because they look the same when they are rotated. Let's examine some shapes with symmetries.



This hexagon has *reflection* symmetry. This means that if it's reflected across a mirror line, the reflection will look exactly the same as the original:



The hexagon has six degrees of rotational symmetry because it can be rotated six times and still appear the same.

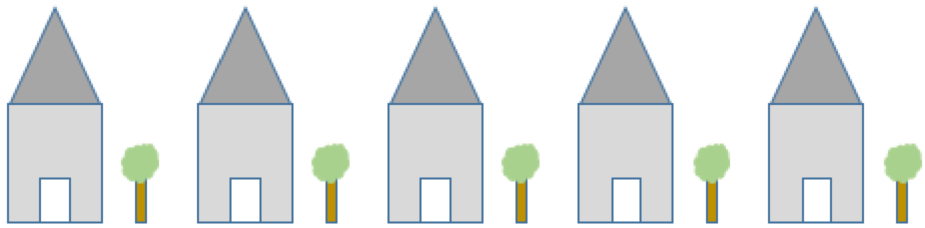


1. If the hexagon has six degrees of rotational symmetry, how many degrees of rotational symmetry does a square have? What about a pentagon?

2. Of the following animals, which one best displays rotational symmetry: lion, octopus or eagle?

3. Draw any shape (does not need to be symmetric) and draw a mirror line next to it. Reflect the shape across the mirror line.

4. Translational symmetry is the property of a shape repeating itself in any direction. The picture below has translational symmetry to the right and left because it is repeating itself over and over.

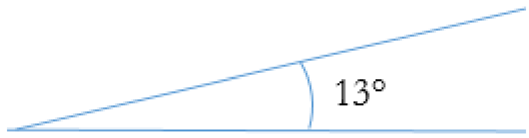


a. Does this picture have any rotational symmetry? What about reflection symmetry?

b. Draw a series of four rectangles such that they have translational symmetry.

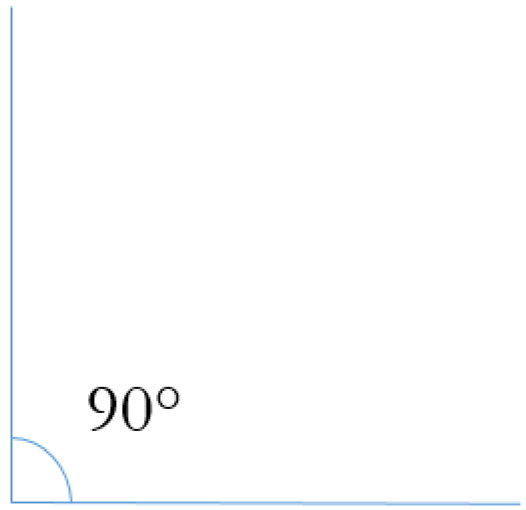
Angles (1 hour)

Angles are formed at the intersection of two lines. The key property of an angle is the number of degrees it has. We use the words *acute*, *obtuse* and *right* to describe these angles. An acute angle is less than 90°. Here's an example of an acute angle:



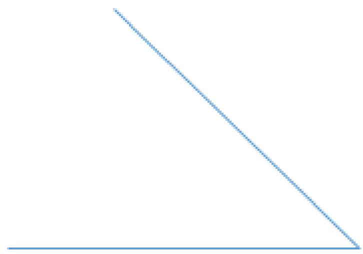
In everyday language we might call this a *sharp* angle. An obtuse angle is the opposite. It is more than 90°.

What if the angle is exactly 90°? This would be a *right* angle and it's a special case—we construct squares and rectangles out of these angles and many of the most common shapes in our daily life are designed around it:



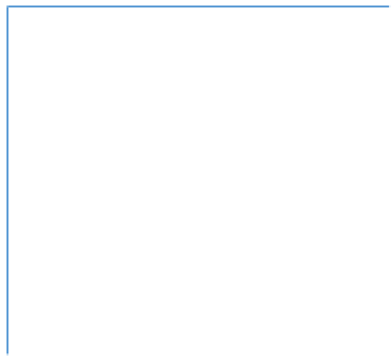
Estimate the measure of the following degrees in angles and describe them as acute, obtuse or right:

1.

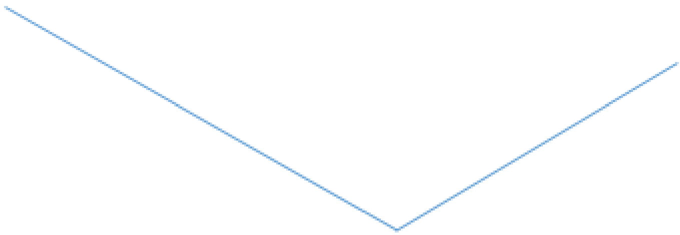


2.

3.



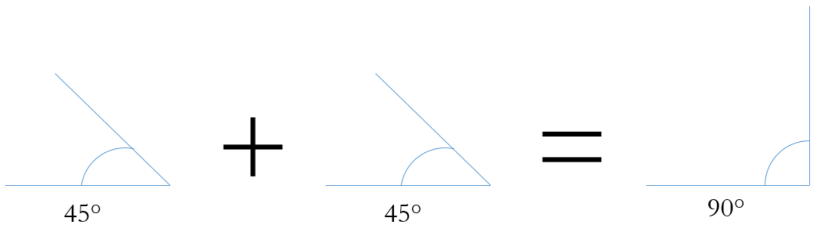
4.



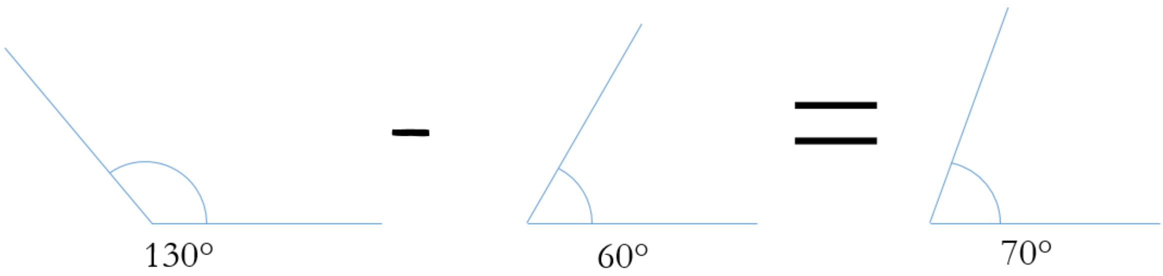
Adding and Subtracting Angles

Since we can represent angles with numbers of degrees, we can also use maths to manipulate them! To add angles, simply add their measure in degrees. To subtract them, just subtract their measure in degrees. The same rules apply for multiplication and division.

$$45° + 45° = 90°$$



$$130° - 60° = 70°$$



Compute and draw the following angles:

5. $30° - 10°$

6. $15° + 45°$

7. $180° - 40° - 40$

8. $9 \times 10°$

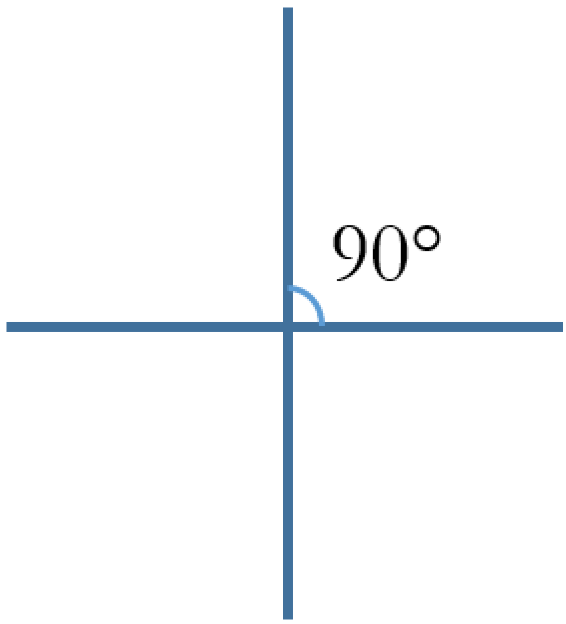
9. $360° - 250°$

Parallel and Perpendicular Lines (30 minutes)

Two lines are parallel if they are both pointing in the exact same direction. That is, even if these lines continue on forever they will never cross.

These lines both have the same angle with any third line that intersects them both.

Perpendicular lines are different; two perpendicular lines intersect and form a 90 degree angle:



Do the following:

10. Draw four parallel lines.

11. Draw two lines that are both perpendicular to a third line.

12. Draw two parallel lines which both have an angle of 45° with a third line

Triangles and Quadrilaterals

As their name suggests, triangles are made of three angles and three sides. The sum of the angles in the triangle must add up to $180°$. If we know the measure of two of the angles, then we can find the third angle by subtracting the first two angles from 180.

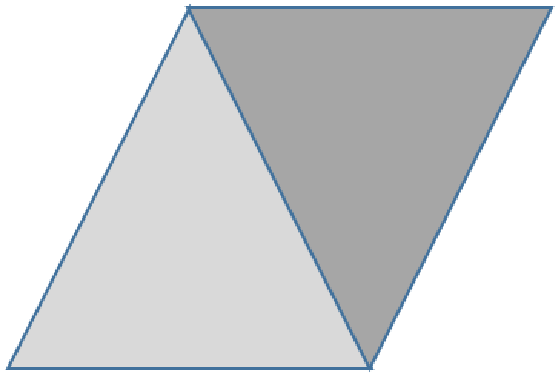
Shapes with four sides or *quadrilaterals* share a similar property. All the angles in a quadrilateral must always add up to 360°. A square, for example, has four angles each of 90°$4 \times 90° = 360°$.

13. A triangle has two angles of sixty degrees. What is the measure of the third angle?

14. Using a protractor, draw a triangle with angles of fifty, sixty and seventy degrees

15. Draw a quadrilateral with an angle of $110°$

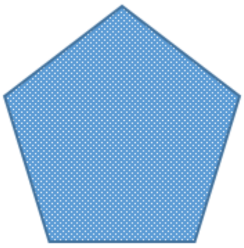
16. Look at the two triangles below. They each have angles which add up to 180°. What is the measure of all their angles combined? What other shape has this angle sum?

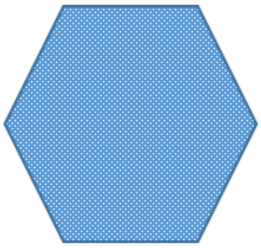


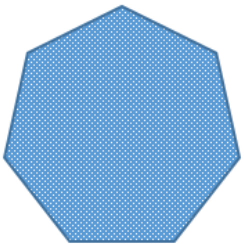
Regular Shapes

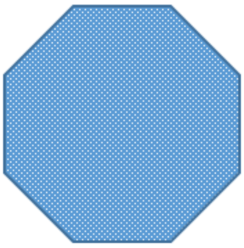
17. Name all of the equilateral shapes shown below:

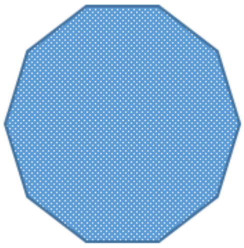
Square

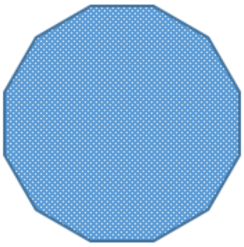












Section 4 – Data and Measures

4.5 hours

Units, Volume and Area (1.5 hours)

1. Sort the following units of measurement onto the table below:

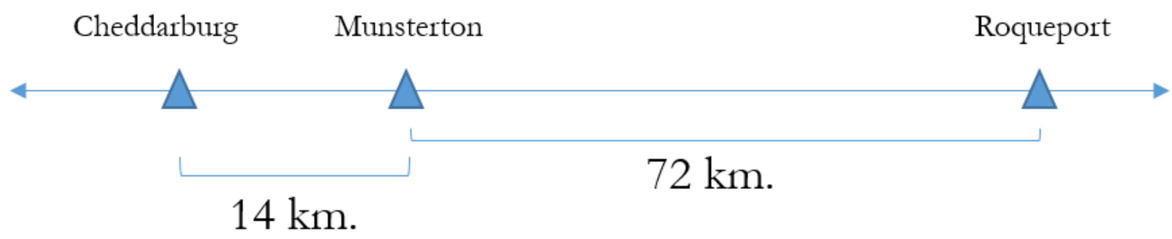
Kilometres	Grams
Kilograms	Meters
Millilitres	Litres

	Smaller	*Larger*
Mass		
Length		
Volume		

2. Calculate the number of millimetres in one kilometre.

3. Write the abbreviations for all of the units listed in exercise 1.

The picture shows the locations of several towns along a road. Use it to answer the following questions:



4. What is the distance from Cheddarburg to Roqueport?

5. If Samantha rides her bike at 20 km. per hour from Cheddarburg to Roqueport, how many hours does it take for her to make the trip?

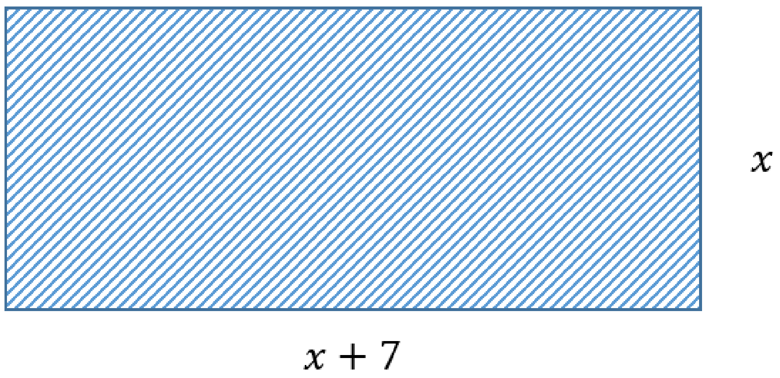
6. Samantha begins her return trip at a speed of 20 km. per hour. While biking back in the other direction from Roqueport to Cheddarburg, Samantha becomes ill near Munsterton. She then finishes the trip at 10 km. per hour from Munsterton to Cheddarburg.
    a. How long does it take for her to read Cheddarburg?

    b. What is her average speed on the return trip?

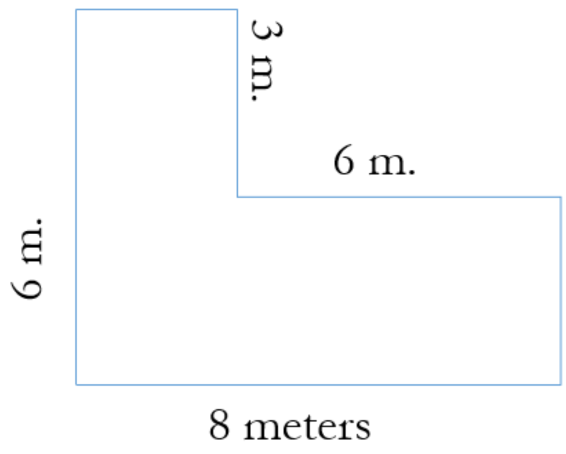
7. How many square centimetres are in one square meter?

8. How many square millimetres are in one square centimetre?

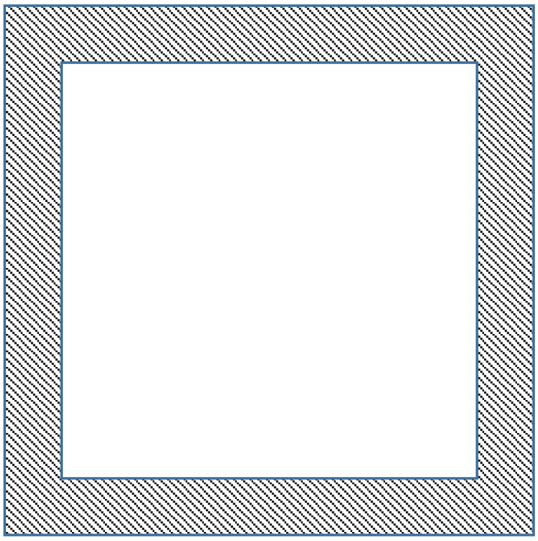
9. The area of the rectangle shown below is 44. Find the value of x and the perimeter of the rectangle.



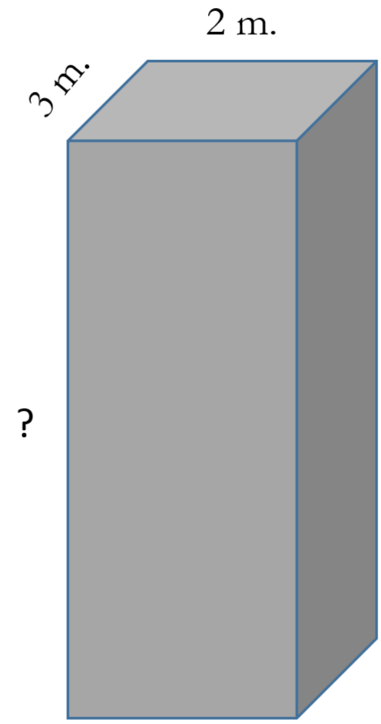
10. Find the area and perimeter of the shape shown below.



11. The shape below consists of an outer square and an inner square. The outer one has side lengths of 7 cm. while the inner has side lengths of 5.5 cm. Find the area of the shaded region.



12. This solid has side lengths of 2 m. and 3 m. Its volume is 60 cubic meters. Find the length of the third side. Use the formula $Volume = Length \times Width \times Height$.

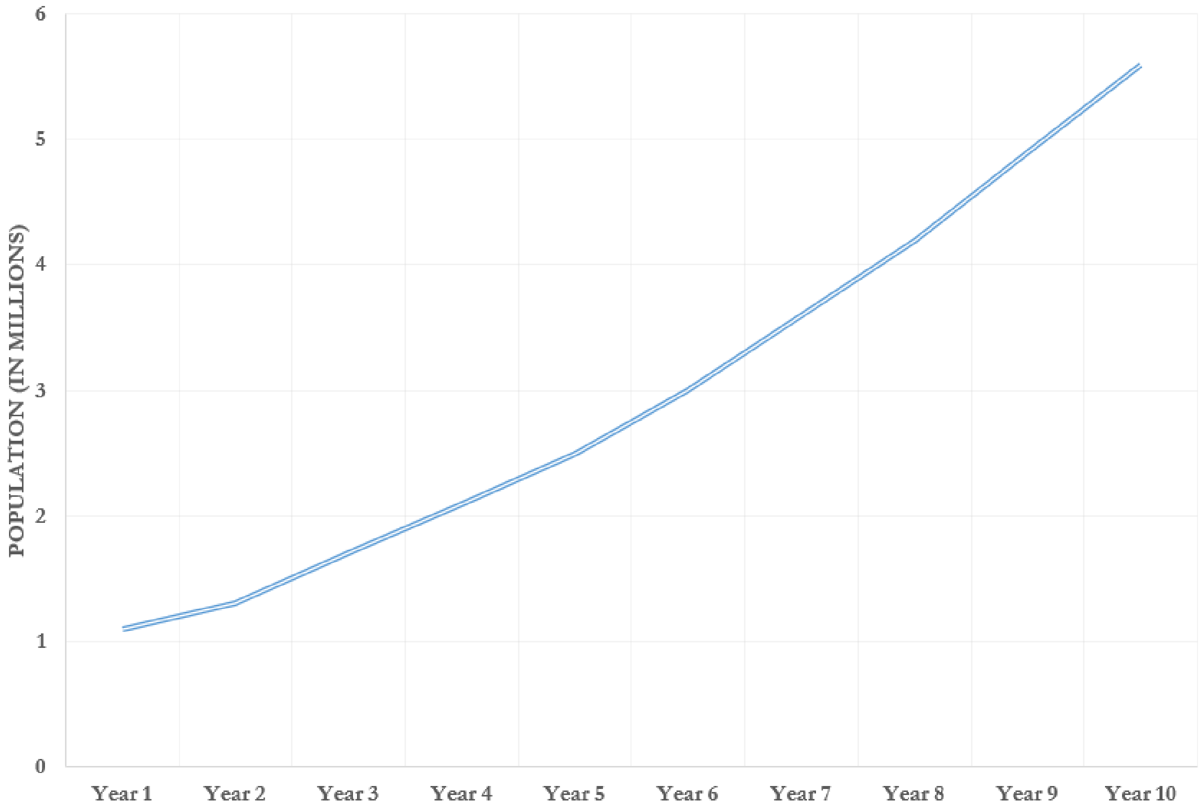


a. Compute the total surface area of this solid.

13. The area of a triangle is given by the formula $area = \frac{height \times width}{2}$. Find the area of a triangle with a height of 12 and a width of 7.

Data (3 hours)

Line Graphs



The line graph above shows the growth of a city's population over a decade. Line graphs connect plot points to show a *trend* over time. Use this one to answer the following questions.

1. What was the approximate population at the beginning of this graph in year 1?

2. What was the final population in year 10?

3. What was the total gain in population?

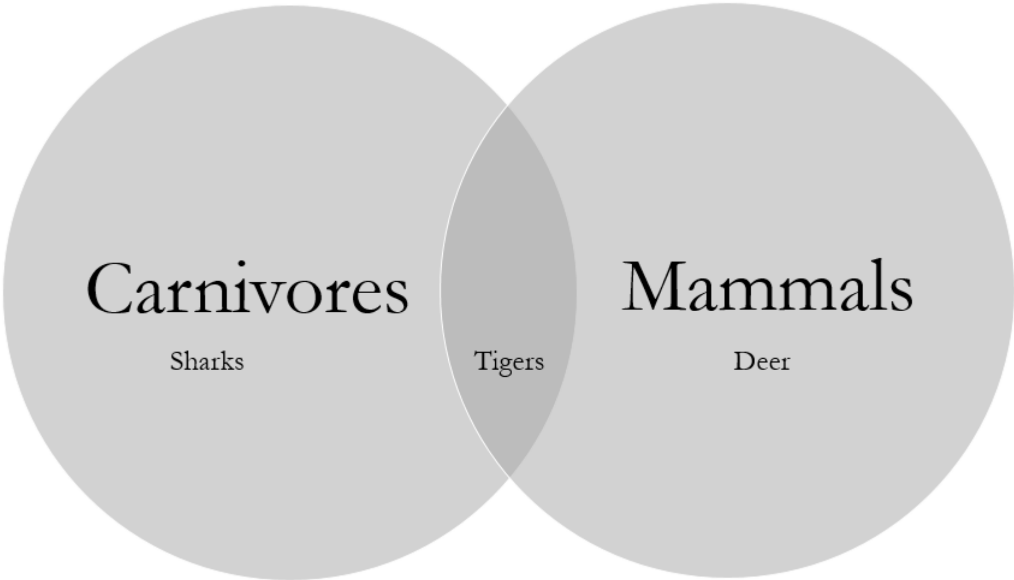
4. A total of 9 years' growth is recorded on this chart. Using your answer to problem 3, find the mean population growth per year.

5. Use this chart to give a rough estimate of what the city's population might be in years 11 and 12.

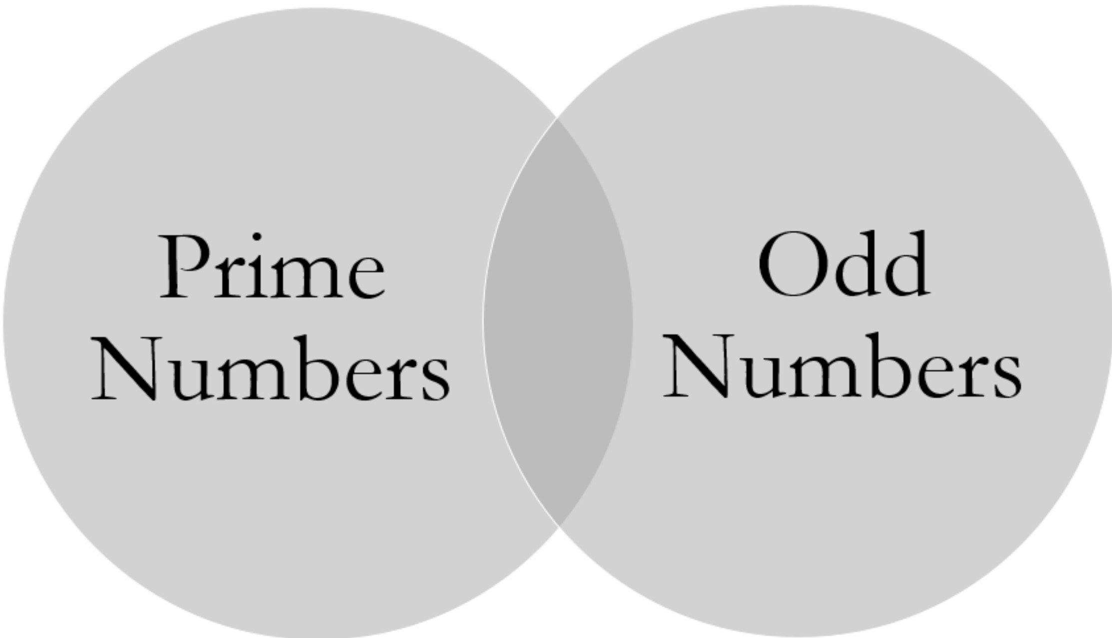
6. Draw a line graph that roughly plots the average temperature against the month for Dubai. Keep in mind that that the temperature is highest in the summer and lowest in the winter.

Venn Diagrams

Venn diagrams are useful for showing groups of objects which might overlap. The chart below is an example of a biology-related Venn diagram. Not all mammals are carnivores nor are all carnivores mammals, but many are.

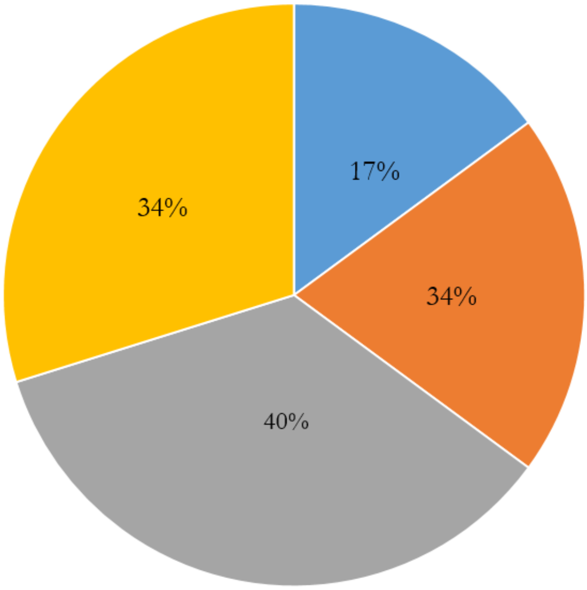


7. Fill in the Venn diagram with the numbers 1-20. Not all of them will need to be shown in diagram.



Pie Charts

Pie charts represent portions of a whole with wedges in a circle. All the wedges must add up to 100% or 1.0 in the decimal scale. Here's an example:



In this pie chart, the percentage that each wedge has of the whole is shown.

8. If this pie chart represents 200 items, what is the number of items corresponding to the 34% wedge?

Pictograms

Representing data with pictures can be an effective way to make a point. This table shows a list of restaurants with their rating on a 1-5 star scale. More stars indicate better reviews.

Restaurant	Rating
Zambroni's	★★★
The Brass Wok	★★
Roadkill Steakhouse	★★★★
Bridge to India	★★
Squishy Freeze	★

9. Which restaurant scored best?

10. If each restaurant can charge customers ten dollars per star it has won, how much can the restaurant *Bridge to India* charge?

Frequency Data

There are three types of averages used to summarize data:

The mean is defined as the sum of a list of numbers divided by how many numbers are in the list.

$$mean\ of\ (0, 1, 2, 3) = \frac{0 + 1 + 2 + 3}{4}$$

The median is defined as the number which is closest to the middle of the list; that is, if you ordered all the numbers by size, the median would be in the middle. If there are two numbers which are both in the middle (this happens in lists with an even number of items) then the median is the mean of those two numbers.

$$median\ of\ (1, 2, 4, 6, 8) = 4$$

$$median\ of\ (6, 4, 5, 3) = \frac{4 + 5}{2} = 4.5$$

Finally, the mode is just the most common number in a list of numbers.

$$mode\ of\ (1,2,2,1,3,5,1) = 1$$

11. Find the mean, median and mode (if it exists) of the following distributions
    a. 1 1 1 2

    b. 10 13 8 7 6 9 17

    c. 2 4 6 8 10 12

    d. 1 2 3 3 97 98 99 100

    e. 1 4 9 16 25 36 49 64

12. A teacher has given two exams. The distribution of the scores of each exam is given below:

Exam 1:	95	89	90	72	55	88	90	82
Exam 2:	85	88	78	91	94	83	99	79

a. Which exam had a greater range of scores?

b. Which exam had a higher mean score?

c. Which exam was harder? Why?

Probability

Measuring the likelihood of events requires an understanding of their probability. We are typically concerned with just one or a few possible events out of all the possibilities. Thus, the formula for probability is as follows:

$$probability = \frac{number\ of\ desired\ outcomes}{total\ number\ of\ outcomes}$$

Let's use a simple example to illustrate what this means. If you flip a coin, the probability of it landing on heads is $\frac{1}{2}$ because there are two possible outcomes (heads or tails) but only one of them that matches what we want.

If we roll a fair and balanced die, each of the numbers 1-6 has an equal probability of being rolled. The probability of rolling a three is $\frac{1}{6}$ while the probability of rolling an even number is $\frac{3}{6}$ because there are three even numbers (2, 4, 6) out of the possibilities (1, 2, 3, 4, 5, 6). We express probabilities in decimal form. Since you have already done conversions from fractions to decimals, finding the decimal value of probabilities is just an application of this.

13. There are three red, two blue and two green marbles in a bag. One marble is pulled out at random. What is the probability of pulling out a blue marble? Express this answer both as a fraction and a decimal.

14. Lotteries make money by giving away huge sums of money at random. If a ticket costs very little in comparison, explain how lotteries can make a profit.

15. Which probability represents a greater likelihood, 0.43 or 0.97?