1. If $x = 16$, which of the following is equal to $\frac{x^4}{2^x}$?

   (a) $\frac{1}{2}$

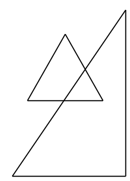
   (b) 1

   (c) $\frac{1}{256}$

   (d) 2

   (e) 16

2. The sum of the areas of two triangles is 120 cm^2. The triangles are placed as shown:



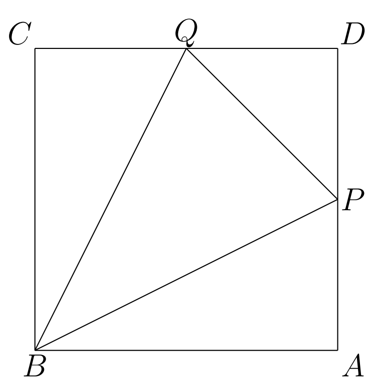
   If $\frac{2}{3}$ of the smaller triangle does not overlap, and $\frac{8}{9}$ of the larger triangle does not overlap, determine the area of the overlap region.

3. If $x = \frac{1}{3}$, $y = \frac{1}{4}$, and $z = \frac{1}{5}$, then determine

$$\frac{xy + xz + yz}{xyz}$$

4. How many numbers in the set $\{10, 11, 12, ..., 98, 99\}$ increase in value when the order of their digits is reversed?

5. In the diagram, square ABCD has side length 4 and P and Q are the midpoints of AD and CD respectively. Determine the area of ΔBPQ.



6. Eleven consecutive integers add together to give 352. Determine the smallest of the eleven integers.

7. We have a set of integers, which contains the integer 2. The average of all the integers is 10. If we remove 2 from the set, the new average is 14. How many integers are there?

8. An arithmetic sequence has 5 terms. The sum of the first two terms is 2 and the sum of the final 2 terms is -18. Determine the third term.

9. The integers 229 and 419 have digits whose product is 36. How many three digit positive integers have digits whose product is 36?

10. If $x^3 + y^3 = 28$ and $x^2 - xy + y^2 = 7$, determine all possible values of $x^2 + y^2$.

11. A **lattice point** is a point (x, y) with x, y both integers. How many lattice points are there above the parabola $y = x^2$ and below the line $y = 100$, including the lattice points on the parabola or line?

12. Determine all real numbers b so that the equations $x^2 + bx + 1 = 0$ and $x^2 + x + b = 0$ have a common solution (at least one solution in common).

13. Player 1 tosses 3 coins and removes all heads. Player 2 tosses the remaining coins. What is the probablity that Player 2 tosses exactly 1 head?

14. Determine the sum of the number of digits in the number 2^{2016} plus the number of digits in the number 5^{2016}

15. A triangle with side lengths 6, 8, and 10 has the property that it's perimeter is equal to it's area, and that it is right angled, and that the side lengths are all integers.

    Which other triangles have all these properties?

16. Let a, b, c, and d be four of the nine numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$.

    There are 4!=24 four digit numbers that may be formed by using each of a, b, c, and d as digits once. Let S be the average of all the numbers.

    Determine the smallest possible value of $a + b + c + d$ so that S is an integer.

17. There are 25 horses and 5 horses race at a time. A horse always run a race in the same time. You cannot record individual times, but can record the order in which the horses finish a race. How many races are needed to determine the 3 fastest horses?

18. The side lengths a, b, and c of ΔABC satisfy the equation $\dfrac{a^3 + b^3 + c^3}{a + b + c} = c^2$. Determine $\angle ACB$.

19. A sequence of 100 numbers x_1, x_2, ..., x_{100} has the property that for each k between 1 and 100 inclusive, x_k is k less than the sum of the other 99 numbers. Determine x_{50}.

20. Let $f(x)$ be an integer polynomial of positive degree. That is, $f(x) = a_n x^n + ... + a_1 x + a_0$ for some integers a_n,..., a_1, a_0 with $n > 0$ and $a_n \neq 0$.

    Prove there exists an integer A so that $f(A)$ is not prime.

21. Ling has a 2×13 hallway that she is going to tile. She has 11 white tiles and 15 black tiles. In how many ways may she tile the hallway so that no two black tiles are adjacent (no two black tiles share an edge)?

For any questions related to mathematics please feel free to contact me at rgarbary@uwaterloo.ca