

Excelling at Mathematics

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Mathematics**

Why do some children excel at Mathematics?

We are not going to answer this question. Instead we will focus upon the following question and some strategies to help.

How do students become better at Mathematics?



The obvious answer ... Practice

But what sort of practice?

Repetitive exercises testing a particular, isolated skill is not the way.

Tip 1: Students must be allowed to struggle upon a problem to make progress.



Two Tasks to Practice Addition and Subtraction of Fractions

Task 1

1. $\frac{3}{7} + \frac{4}{7} =$ _____

2. $\frac{1}{2} + \frac{2}{11} =$ _____

3. $\frac{2}{4} + \frac{3}{12} =$ _____

4. $\frac{4}{5} + \frac{1}{4} =$ _____

5. $\frac{3}{8} + \frac{1}{8} =$ _____

6. $\frac{2}{7} + \frac{3}{5} =$ _____

7. $\frac{4}{6} + \frac{3}{10} =$ _____

8. $\frac{4}{10} + \frac{5}{8} =$ _____

9. $\frac{2}{3} + \frac{2}{3} =$ _____

10. $\frac{2}{11} + \frac{1}{2} =$ _____

Task 2

Unit fractions (fractions which have numerators of 1) can be written as the sum of two different unit fractions.

For example

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$$

Charlie thought he'd spotted a rule and made up some more examples.

$$\frac{1}{2} = \frac{1}{10} + \frac{1}{20}$$

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

$$\frac{1}{3} = \frac{1}{7} + \frac{1}{21}$$

$$\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$$

Are all his examples correct?

What do you notice about the sums that are correct?

Find some other correct examples..

How would you explain to Charlie how to generate lots of correct examples?



Time for some Mathematics

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Why was this a superior task to the worksheet on addition of fractions?

It allowed practice of adding and subtracting.

Allowed opportunities to look for patterns

Opportunities to make and test the students own conjectures.



Inner and Outer Tasks

In a worksheet or textbook exercise, all attention is on answering the question.

Shift Attention from key skill being practiced.



Generate 5 random integers from 1 to 9

Put them in the boxes below to make the biggest product.

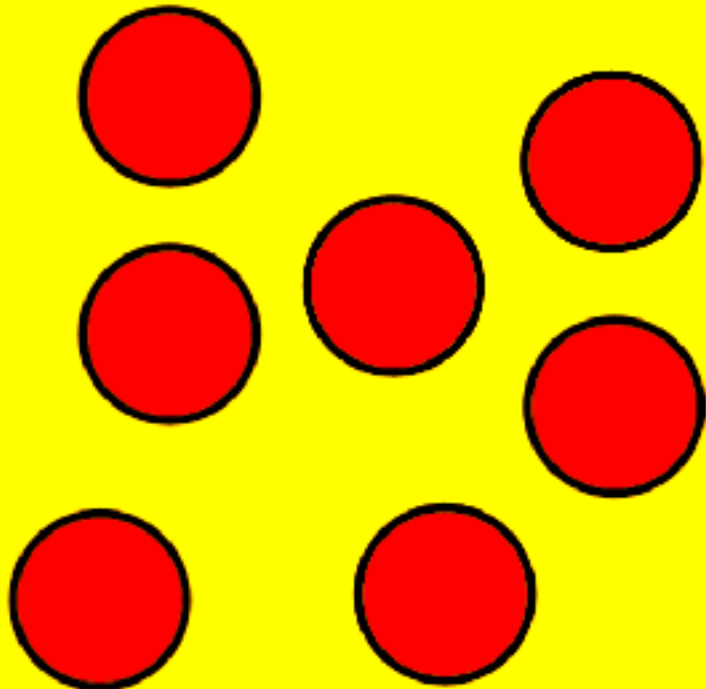
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This can be turned into a game by playing against a partner and having to put a number in a box as it is chosen.



Now Time for a Game

Take it in turns to remove 1 or 2 counters. The loser is the person who is left with 1.



Questions

- Can one player always win?
- Can you devise a winning strategy



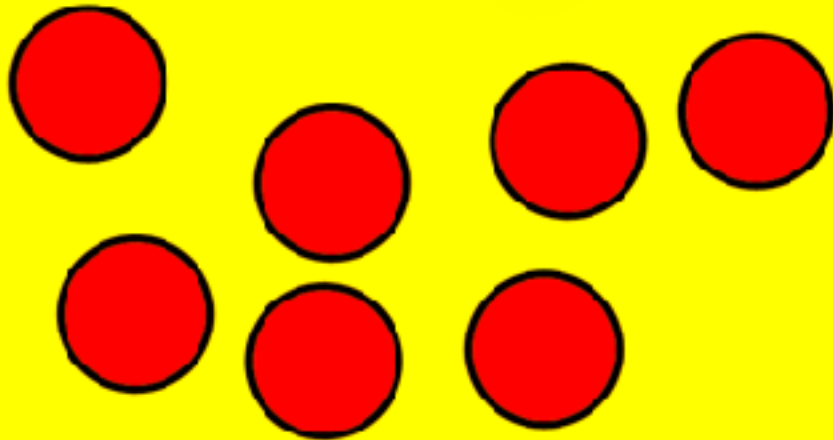
Now Time for a Game

Now change the number of starting counters to 8 and 9.

Does the same strategy still work? What about 10?

Now extend the game by being able to take a different number of counters away.

Can you generalise what you have found?



- Can you find a general rule for always winning games of this type.

- When can player 1 always win and when can player 2?





Challenge the teacher!

You get to choose

1. How many counters we start with.
2. How many can be taken away each time

I choose whether to go first or second

TIP 2: Play the games your child has played in Maths class with them. Also let them win sometimes, though you may find that you can't beat them.



Flipping the question

A very common question related to quadratic equations could be..

Find the y-intercept and x-intercept of the graph with the following equation $y = x^2 + 5x + 6$

Instead this could be reworded to..

How many different quadratic equations with integer coefficients can you find with y-intercept (0, 6) and one root of $x = -3$



Generalisations

Mathematics is full of generalisations and one thing all these tasks had in common is that they allow students the opportunity to generalise.

$$3 + 2 = 5$$

$$\frac{3}{17} + \frac{2}{17} = \frac{5}{17}$$

$$3x + 2x = 5x$$



Practice through Progress

Incorporate previous topic in new topic being covered.

Enables continued practice of a topic not mastered alongside being able to learn new material.

1. Complete the table of values for the graph

$$y = \frac{2}{3}x + 2$$

x	-2	$-\frac{4}{5}$	0	$\frac{3}{4}$	$1\frac{1}{2}$	2
y						

2. The four lines below form the boundary of a parallelogram. Draw the lines and work out the area of the parallelogram:

$$y = 2x + 3\frac{2}{5} \qquad y = 4$$

$$y = 2x - 1 \qquad y = 1\frac{4}{7}$$

3. Find the equations of 4 lines which bound a parallelogram with the area of eleven and five-eighths



Misconceptions

Identifying why a student got a particular problem wrong.

What mistake has been made in the following questions? Can you give the correct answer?

Question

Answer

1) Factorise $4x^2 + 12x$

$$4x(x + 12)$$

2) Expand and simplify $(x + 3)(x - 2)$

$$x^2 - 6$$

3) Simplify $2a^3 \times 3a^2$

$$5a^5$$



Time for a Magic trick

Choose a 3 digit number, where the digits are all different.

Switch the hundreds and unit digits and subtract the smaller number from the larger

With your answer switch the hundreds and units digits again, then add the 2 results.

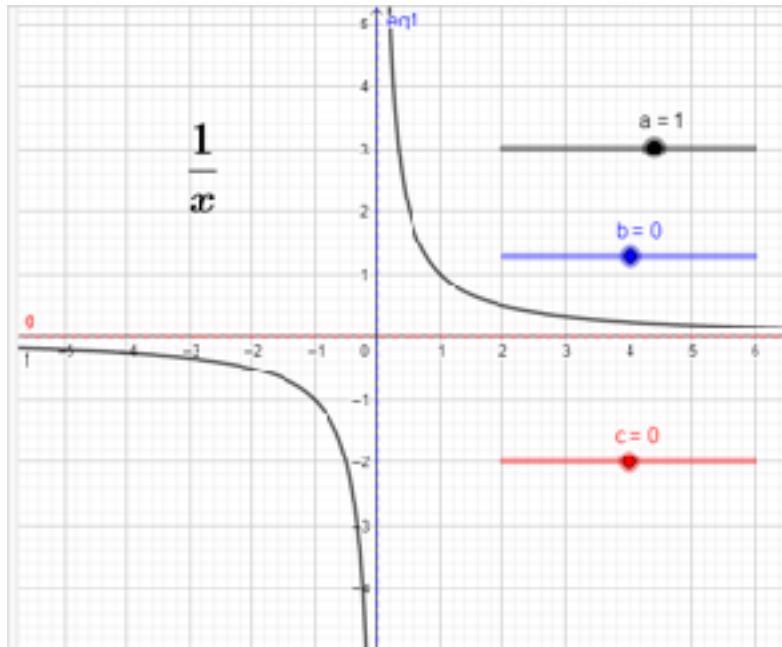
Tell me your number.



My favourite websites

GeoGebra

<https://www.geogebra.org/u/nicholasturner271>



Some More Games

Factors and Multiples

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

- Rules**
- 2 players take it in turns crossing out 1 number from the board.
 - The 1st player **must** choose an **even** number.
 - After this a player **must** choose either a factor of multiple of the previous choice
 - The first player to not be able to cross a number out loses.

Divisibility Tests

Generate 2 random integers from 1 to 9 and choose one of your own. The aim is to make the biggest;

1. Multiple of 2
2. Multiple of 3
3. Multiple of 4
4. Multiple of 6



**Thank you :)
Questions?**

