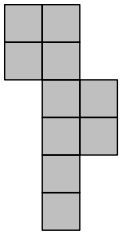
3 points

 # 1. This shape is made from ten squares of side length 1 cm. What is its perimeter?

下图是一个由十个边长为1 cm的正方形所组成的形状。请问该形状的周长是多少?



(**A**) 14 cm (**B**) 18 cm (**C**) 30 cm (**D**) 32 cm (**E**) 40 cm

---

 # 2. When the following five sums are arranged in order from smallest to largest, which one will be in the middle?

计算以下几个选项的值，然后按从最小到最大顺序排列，请问哪一个会出现在中间?

(**A**) $1 + 2345$ (**B**) $12 + 345$ (**C**) $123 + 45$ (**D**) $1234 + 5$ (**E**) 12345

 # 3. Who is the mother of the daughter of Anne's mother's mother?

请问Anne的妈妈的妈妈的女儿的妈妈是谁?

(**A**) Anne's sister
Anne的姐姐

(**B**) Anne's niece
Anne的侄女

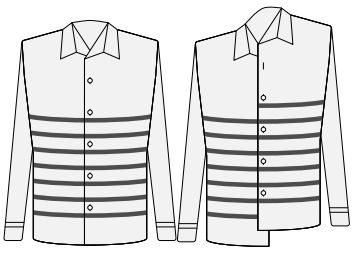
(**C**) Anne's mother
Anne的妈妈

(**D**) Anne's aunt
Anne的阿姨

(**E**) Anne's grandmother
Anne的外婆/姥姥

 # 4. When Cosmo wears his new shirt properly as shown on the left, the horizontal stripes form seven closed rings around his waist. This morning he buttoned his shirt wrongly, as shown on the right. How many closed rings were there around Cosmo's waist this morning?

如下方左图所示，当Cosmo正确的扣上新衬衫的扣子时，衬衫的条纹在他的腰部形成了七个闭合的圈。而右图显示的是他今天早上不小心把扣子扣错了的状况。请问Cosmo今早的衬衫条纹在他腰间形成了多少个闭合的圈?



(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

 # 5. In the calculations below each letter stands for a digit. What is the total of the four numbers on the right?

以下所示的算式中，每个字母代表一个数字。问右侧的四个数的总和是多少?

	A D
	+ C D
A B	+ A B
+ C D	+ C B
7 9	?

(**A**) 79 (**B**) 158 (**C**) 869 (**D**) 1418 (**E**) 7979

 # 6. The sum of four consecutive integers is 2. What is the smallest of these integers?

四个连续整数的总和为2。这些整数中最小的整数是多少?

(**A**) -3 (**B**) -2 (**C**) -1 (**D**) 0 (**E**) 1

 # 7. The years 2020 and 1717 both consist of a two-digit number repeated twice. How many years after 2020 will it be until the next year which has this property?

2020年和1717年均由重复两次的两位数组成。请问下一个拥有同样特质的年份会在几年后出现?

(**A**) 20 (**B**) 101 (**C**) 120 (**D**) 121 (**E**) 202

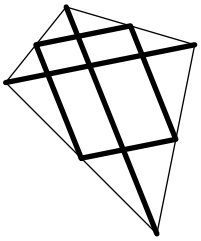
 # 8. Mary has ten pieces of paper. Some of these are squares and the rest are triangles. She cuts three squares diagonally from corner to corner. She counts the total number of vertices of the 13 pieces of paper she has now, and gets the answer 42. How many triangles did she have before making the cuts?

Mary有十张纸。其中一些是正方形，其余的是三角形。她将三个正方形沿着对角线剪开。过后，她计算了13张纸的顶点总数，得到的答案为42。请问在裁剪之前，她有多少个三角形?

(**A**) 8 (**B**) 7 (**C**) 6 (**D**) 5 (**E**) 4

 # 9. Martin made a kite by cutting a straight wooden pole into 6 pieces. He used two of them, of lengths 120 cm and 80 cm, as the diagonals. The remaining four pieces connected the midpoints of the sides of the kite, as shown. How long was the pole before it was cut?

Martin将一根笔直的木棍切成6根来制作风筝。他使用其中两根长度分别为120 cm和80 cm的木棍来当成风筝的对角线，如图所示，其余四根木棍则连接着风筝四条边上的中心点。请问木棍原本的长度是多少?

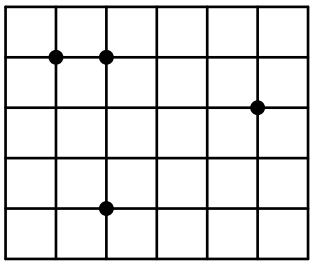


(**A**) 300 cm (**B**) 370 cm (**C**) 400 cm (**D**) 410 cm (**E**) 450 cm

---

 # 10. Four points are marked on a grid of squares of side-length 1. Forming a triangle using three of these points, what is the smallest area that can be obtained?

有四个点被标记在边长为1的方格上。用其中三个点组成一个三角形，能得到的最小面积是多少?



(**A**) $\frac{1}{2}$ (**B**) 1 (**C**) $\frac{3}{2}$ (**D**) 2 (**E**) $\frac{5}{2}$

---

4 points

 # 11. Helen wants to spend 18 consecutive days visiting her Grandma. Her Grandma reads her story books on story days Tuesday, Saturday and Sunday. Helen wants to spend as many story days with her Grandma as possible. On which day of the week should she start her visit?

Helen想连续18天都去看望她的奶奶。星期二、星期六和星期天是故事日，她的奶奶会在这几天给她讲故事，如果Helen想尽可能多地在故事日陪伴奶奶，请问她应该在一周中的哪一天开始去看奶奶?

(**A**) Monday 周一 (**B**) Tuesday 周二 (**C**) Friday 周五 (**D**) Saturday 周六 (**E**) Sunday 周日

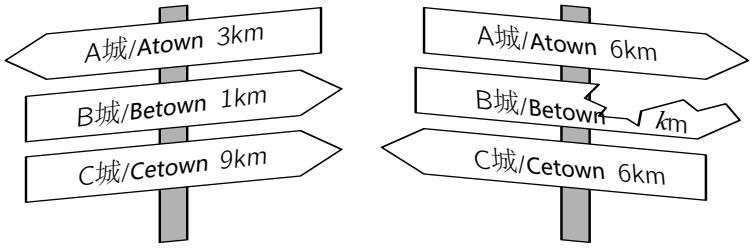
 # 12. The integers a, b, c and d satisfy $ab = 2cd$. Which of the following numbers could not be the value of the product $abcd$?

整数a、b、c和d能够满足ab = 2cd。以下哪个数字不可能是abcd的乘积?

(**A**) 50 (**B**) 100 (**C**) 200 (**D**) 450 (**E**) 800

 # 13. The shortest path from Atown to Cetown runs through Betown. Walking on this path from Atown to Cetown, we would first find the signpost shown on the left. Later we would find the signpost shown on the right. What distance was written on the broken sign?

从A城到C城最短的路程是经过B城。沿着从A城到C城的路线走，我们首先会看到如下方左侧显示的路牌。继续行走一段时间后，我们将会看到如下方右侧显示的路牌。请问路牌上破损的地方写着的距离是多少?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

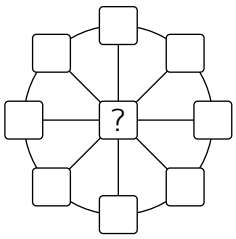
 # 14. An isosceles triangle has a side length equal to 2/5 of another side length. The third side has length 20 cm. What is the perimeter of this triangle?

一个等腰三角形的其中一条边长是另一条边长的2/5，该等腰三角形的第三条边长为20cm。请问该等腰三角形的周长是多少?

(**A**) 36 cm (**B**) 48 cm (**C**) 60 cm (**D**) 90 cm (**E**) 120 cm

 # 15. Tom wants to write a number in each of the nine boxes in the figure shown. He wants the sum of the three numbers on each diameter to be 13 and the sum of the eight numbers on the circumference to be 40. What is the number that Tom has to write in the central cell?

Tom想在下图所示的9个方格中各写一个数字。他希望每个直径上的三个数字的总和为13，而圆周上八个数字的总和为40。Tom应该在中间的方格中填写哪个数字?



(**A**) 3 (**B**) 5 (**C**) 8 (**D**) 10 (**E**) 12

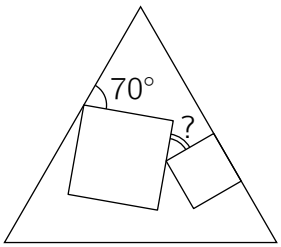
 # 16. Masha put a multiplication sign between the second and the third digits of the number 2020 and noted that the resulting product $20 \times 20 = 400$ is a perfect square. How many numbers between 2010 and 2099 (including 2020) have the same property?

Masha在数字2020的第二和第三个位数之间放了一个乘号，所得的乘积20x20=400是一个完全平方数。请问从2010年至2099年（包括2020年）之间有多少个数字具有相同属性?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

 # 17. Two squares of different size are drawn inside an equilateral triangle. What is the size of the angle marked by the question mark?

在等边三角形内绘制两个大小不同的正方形，请问标记问号的角度是多少?



(**A**) 25° (**B**) 30° (**C**) 35° (**D**) 45° (**E**) 50°

 # 18. Luca began a 520 km car trip with 14 liters of fuel in the tank. His car consumes 1 liter of fuel per 10 km. After driving 55 km, he reads a road sign showing the distances to five petrol stations ahead on the road, which are 35 km, 45 km, 55 km, 75 km and 95 km. The capacity of the fuel tank is 40 liters and Luca wants to stop just once to fill the tank. How far is the petrol station that he should stop at?

Luca想要完成一段520km的旅程。他的油箱里装了14升的燃油。然而，他的汽车每10km将消耗1升的燃油。行驶55km后，他看见了一个路标，上面写着前方5个加油站的距离，分别为35km、45km、 55km、75km和95km。汽车的燃油箱容量为40升，而Luca只想停在一个加油站然后加满油箱。那么他应该停在多远的加油站?

(**A**) 35 km (**B**) 45 km (**C**) 55 km (**D**) 75 km (**E**) 95 km

---

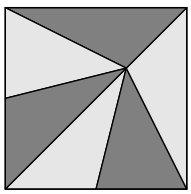
 # 19. Let $17x + 51y = 102$. What is the value of $9x + 27y$?

若17x + 51y = 102，那么9x + 27y的值是多少?

(**A**) 54 (**B**) 36 (**C**) 34 (**D**) 18
(**E**) Cannot be determined 该值不确定

 # 20. A square shaped stained glass window of 81 dm^2 is made out of six triangles of equal area (see figure). A fly is sitting exactly on the spot where the six trangles meet. How far from the bottom of the window is the fly sitting?

下图显示了一个由六个面积相等的三角形组成81 dm^2 的正方形彩色玻璃窗。一只苍蝇正好停在六个三角形相交的地方。请问苍蝇此时的位置到窗户底部的距离是多少?



(**A**) 3 dm (**B**) 5 dm (**C**) 5.5 dm (**D**) 6 dm (**E**) 7.5 dm

5 points

 # 21. The digits from 1 to 9 are randomly arranged to make a 9-digit number. What is the probability that the resulting number is divisible by 18?

把1至9的数字随机排列成一个9位数。请问这个数能被18整除的概率是多少?

(**A**) $\frac{1}{2}$ (**B**) $\frac{4}{9}$ (**C**) $\frac{5}{9}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{3}{4}$

 # 22. A hare and a tortoise competed in a 5 km race along a straight line. The hare is five times faster than the tortoise. The hare mistakenly started perpendicular to the route. After a while he realized his mistake, then turned and ran straight to the finish point. He arrived at the same time as the tortoise. What is the distance between the hare's turning point and the finish point?

一只野兔和一只乌龟沿着一条直线进行了5km的赛跑。野兔的速度是乌龟的五倍。可惜，那只野兔一开始错误地沿着跑道的垂直方向出发，过了一会儿，它才意识到自己的错误，然后再转身直奔终点。野兔和乌龟同时到达终点。请问野兔的转折点和终点之间的距离是多少?

(**A**) 11 km (**B**) 12 km (**C**) 13 km (**D**) 14 km (**E**) 15 km

---

 # 23. There are some squares and triangles on the table. Some of them are blue and the rest are red. Some of these figures are large and the rest are small. We know the following two facts are true:

桌子上有一些正方形和三角形。有些是蓝色的，其余的是红色的。这些图形有的大，有的小。我们知道以下两个事实是正确的:

1) if the figure is large then it is a square and
2) if the figure is blue then it is a triangle.
Which of the statements A–E must be true?

1)如果这个图形很大，那么这个图形是正方形
2)如果这个图形是蓝色的，那么这个图形是三角形
以下A-E的说法中，哪个是正确的?

(**A**) All red figures are squares.
所有红色的图形是正方形

(**B**) All squares are large.
所有的正方形都是大图形

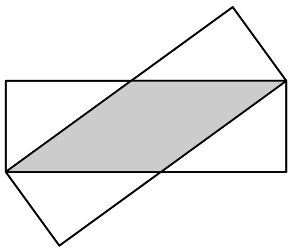
(**C**) All small figures are blue.
所有的小图形都是蓝色的

(**D**) All triangles are blue.
所有的三角形都是蓝色的

(**E**) All blue figures are small.
所有的蓝色图形都是小图形

 # 24. Two identical rectangles with side lengths 3 cm and 9 cm overlap, as shown in the diagram. What is the area of the overlap of the two rectangles?

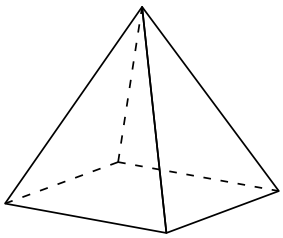
如图所示，两个相同的矩形，边长分别为3 cm 和9 cm，将这两个矩形叠放在一起。请问这两个矩形重叠的面积是多少?



(**A**) 12 cm^2 (**B**) 13.5 cm^2 (**C**) 14 cm^2 (**D**) 15 cm^2 (**E**) 16 cm^2

 # 25. Kanga labelled the vertices of the square-based pyramid using 1, 2, 3, 4 and 5 once each. For each face Kanga calculated the sum of the numbers on its vertices. Four of these sums are 7, 8, 9 and 10. What is the sum of the numbers at the the vertices of the fifth face?

Kanga在一个以正方形为底的金字塔的各个顶点上分别标记了1、2、3、4和5。然后，Kanga计算每个面上的顶点的数字总和，其中四面分别是7、8、9和10。请问第五面上的顶点的数字总和是多少?



(**A**) 11 (**B**) 12 (**C**) 13 (**D**) 14 (**E**) 15

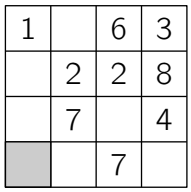
 # 26. A large cube is built using 64 smaller identical cubes. Three of the faces of the large cube are painted. What is the maximum possible number of small cubes that have exactly one face painted?

一个大立方体是由64个小的相同的立方体组成的。这个大立方体的三个面被涂上了颜色。请问最多会有多少个小立方体只有一个面被涂上颜色?

(**A**) 27 (**B**) 28 (**C**) 32 (**D**) 34 (**E**) 40

 # 27. Anna wants to write a number in each square of the grid so that the sum of the four numbers in each row and in each column are the same. She has already written some numbers, as shown. What number does she write in the shaded square?

Anna想在每个正方格内写一个数字，使得每行和每列的四个数字的总和都相同。如下图所示，Anna已经在正方格中填了一些数字。请问她应在阴影正方格内填上什么数字?



(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

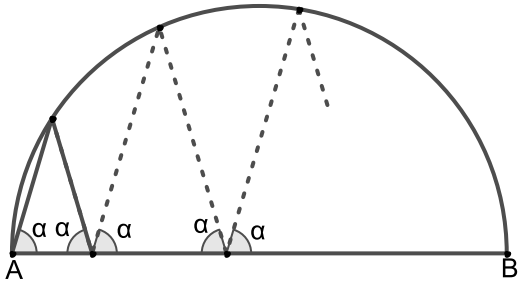
 # 28. Alice, Belle and Cathy had an arm-wrestling contest. In each game two girls wrestled, while the third rested. After each game, the winner played the next game against the girl who had rested. In total, Alice played 10 times, Belle played 15 times and Cathy played 17 times. Who lost the second game?

Alice, Belle 和 Cathy进行了扳手腕比赛。在每一场比赛中，两个人比赛，第三个人休息。每一场比赛后，获胜者与刚才休息的人进行下一场比赛。Alice总共玩了10次，Belle玩了15次，Cathy玩了17次。请问第二场比赛谁输了?

(**A**) Alice
(**B**) Belle
(**C**) Cathy
(**D**) either Alice or Belle could have lost the second game /Alice或Belle会输
(**E**) either Belle or Cathy could have lost the second game/Belle或Cathy会输

 # 29. A zig-zag line starts at point A, at one end of the diameter AB of a circle. Each of the angles between the zig-zag line and the diameter AB is equal to α, as shown. After four peaks, the zig-zag line ends at the point B. What is the size of angle α?

折线从直径AB的端点A开始，如图所示，折线与直径AB之间所形成的每一个角度都等于α。此折线碰到圆的边四次后，在B点结束。请问角度α的值是多少?



(**A**) 60° (**B**) 72° (**C**) 75° (**D**) 77.5° (**E**) 80°

 # 30. Eight consecutive three-digit positive integers have the following property: each of them is divisible by its last digit. What is the sum of the digits of the smallest of the eight integers?

有八个连续的三位数正整数具有这个特点：每个整数均可被其最后一个位数的数字整除。在这八个整数当中，请问值最小的整数的每个位数的总和是多少?

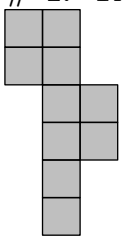
(**A**) 10 (**B**) 11 (**C**) 12 (**D**) 13 (**E**) 14

END OF PAPER

-Scratch Paper-
草稿纸

3 points

 # 1. The diagram shows a shape made from ten squares of side length 1 cm joined edge to edge.



What is the length of its perimeter?

(**A**) 14 cm (**B**) 18 cm (**C**) 30 cm (**D**) 32 cm (**E**) 40 cm

Solution: The perimeter consists of 18 edges of squares and is equal to 18 cm.

 # 2. When the answers to the following calculations are put in order from smallest to largest, which will be in the middle?

(**A**) $1 + 2345$ (**B**) $12 + 345$ (**C**) $123 + 45$ (**D**) $1234 + 5$ (**E**) 12345

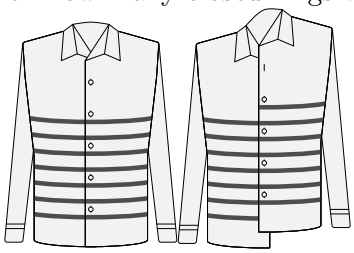
Solution: The expressions in the answers are equal to 2346, 357, 168, 1239, 12345. Therefore the number in the middle is equal to $1234 + 5$.

 # 3. Who is the mother of the daughter of Anne's mother's mother?

(**A**) Anne's sister (**B**) Anne's niece (**C**) Anne's mother
(**D**) Anne's aunt (**E**) Anne's grandmother

Solution: The mother's mother is called grandmother.

 # 4. When Cosmo wears his new shirt properly as shown on the left, the horizontal stripes form seven closed rings around his waist. This morning he buttoned his shirt wrongly, as shown on the right. How many closed rings were there around Cosmo's waist this morning?



(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

Solution: There are no closed rings. The obtained curve is a spiral.

 # 5. In the calculations shown each letter stands for a digit. They are used to make some two-digit numbers. The two numbers on the left have a total of 79. What is the total of the four numbers on the right?

$$\begin{array}{rr} & \mathbf{AD} \\ & +\,\mathbf{CD} \\ \mathbf{AB} & +\,\mathbf{AB} \\ +\,\mathbf{CD} & +\,\mathbf{CB} \\ \hline \mathbf{79} & ? \end{array}$$

(**A**) 79 (**B**) 158 (**C**) 869 (**D**) 1418 (**E**) 7979

SOLUTION: $\overline{AD}+\overline{CD}+\overline{AB}+\overline{CB} = 10(A+C+A+C)+(D+D+B+B) = 20(A+D)+2(B+D) = 2\times(10A+B)+2\times(10C+D) = 2(\overline{AB}+2\overline{CD}) = 2\times 79 = 158$

 # 6. The sum of four consecutive integers is 2. What is the smallest of these integers?

(**A**) -3 (**B**) -2 (**C**) -1 (**D**) 0 (**E**) 1

SOLUTION: If x is the smallest number, then the sum equals $x+(x+1)+(x+2)+(x+3) = 4x+6 = 2$, $x = -1$.

 # 7. The years 2020 and 1717 both consist of a two-digit number repeated twice. How many years after 2020 will it be until the next year which has this property?

(**A**) 20 (**B**) 101 (**C**) 120 (**D**) 121 (**E**) 202

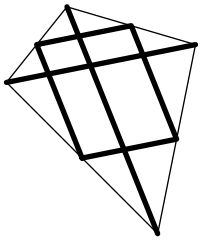
SOLUTION: The smallest year with such property after 2020 with such property equals 2121. $2121 - 2020 = 101$.

 # 8. Mary has ten pieces of paper. Some of these are squares and the rest are triangles. She cuts three squares diagonally from corner to corner. She counts the total number of vertices of the 13 pieces of paper she now has and gets the answer 42. How many triangles did she have before making the cuts?

(**A**) 8 (**B**) 7 (**C**) 6 (**D**) 5 (**E**) 4

SOLUTION: If all of 13 pieces of paper will be triangles, then Mary would obtain $13\cdot 3 = 39$ vertices. Therefore there were $42-39 = 3$ squares and $13-3 = 10$ triangles. $10-3\cdot 2 = 4$ triangles where in the begining.// Another solution: Let t be the number of initial triangles so the squares were initially 10-t. After cutting 3 squares to make 6 extra triangles, we have 7-t squares and t+6 triangles. Counting vertices 4(7-t)+3(t+6)=42, so t=4.

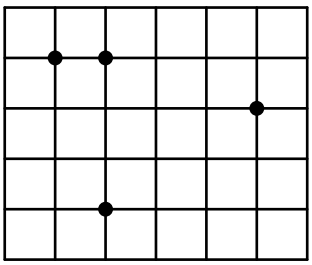
 # 9. Martin made a kite by cutting a straight wooden pole into 6 pieces. He used two of them, of lengths 120 cm and 80 cm, as the diagonals. The remaining four pieces connected the midpoints of the sides of the kite, as shown. How long was the pole before it was cut?



(**A**) 300 cm (**B**) 370 cm (**C**) 400 cm (**D**) 410 cm (**E**) 450 cm

SOLUTION: The pieces that connect the midpoints are two times smaller than the respective diagonals. Therefore the slat before cut was $(120+80)+\frac{120+80}{2}+\frac{120+80}{2} = 400$ cm.

 # 10. Four points are marked on a grid of squares of side-length 1. Forming a triangle using three of these points, what is the smallest area that can be obtained?



(**A**) $\frac{1}{2}$ (**B**) 1 (**C**) $\frac{3}{2}$ (**D**) 2 (**E**) $\frac{5}{2}$

SOLUTION: Any drawn triangle has the width of at least 1 and the height of at least 1. Therefore the area of any triangle is at least $\frac{1}{2} \cdot 1 \cdot 1 = 1/2$. The example is obtained by connecting the three highest marked points in the picture.

4 points

 # 11. Helen wants to spend 18 consecutive days visiting her Grandma. Her Grandma reads her story books on story days Tuesday, Saturday and Sunday. Helen wants to spend as many story days with her Grandma as possible. On which day of the week should she start her visit?

(**A**) Monday (**B**) Tuesday (**C**) Friday (**D**) Saturday (**E**) Sunday

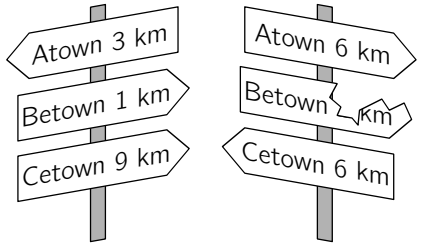
SOLUTION: A seven day period is, of course periodic, so concentrate on the 18-14=4 remaining days. A start on Saturday contains all story days, so it is maximum.

 # 12. The integers a, b, c and d satisfy $ab = 2cd$. Which of the following numbers could not be the value of the product $abcd$?

(**A**) 50 (**B**) 100 (**C**) 200 (**D**) 450 (**E**) 800

SOLUTION: Condition gives $abcd = 2(cd)^2$ (twice a perfect square). Of the numbers given the only one not twice a perfect square is 100. Note that we can find examples to show that all other answers are possible.

 # 13. The shortest path from Atown to Cetown runs through Betown. Walking on this path from Atown to Cetown, we would first find the signpost shown on the left. Later we would find the signpost shown on the right. What distance was written on the broken sign?



(**A**) 1 km (**B**) 2 km (**C**) 3 km (**D**) 4 km (**E**) 5 km

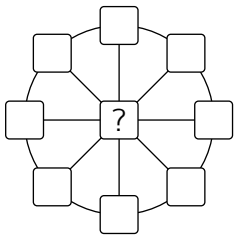
SOLUTION: From both signposts you get the information, that the distance between Atown and Cetown is 12 km. The left signpost shows that the distance between Atown and Betown is 4 km. So the distance between Betown and Cetown is 8 km and the missing distance is 2 km.

 # 14. An isosceles triangle has a side of length 20 cm. Of the other two side lengths, one is equal to 2/5 of the other. Which of the following values is the perimeter of this triangle?

(**A**) 36 cm (**B**) 48 cm (**C**) 60 cm (**D**) 90 cm (**E**) 120 cm

SOLUTION: To fulfill the property of being isosceles and the given equation there are two possibilities: 20 cm, 8 cm, 20 cm and 20 cm, 20 cm, 50 cm. The second does not fulfill the triangle inequality. So the perimeter must be 48 cm.

 # 15. Tom wants to write a number in each of the nine cells of the figure shown. He wants the sum of the three numbers on each diameter to be 13 and the sum of the eight numbers on the circumference to be 40. What number has Tom to write in the central cell?



(**A**) 3 (**B**) 5 (**C**) 8 (**D**) 10 (**E**) 12

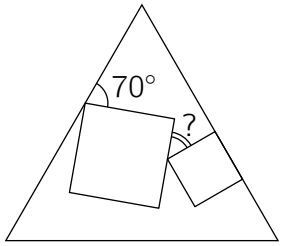
SOLUTION: Adding 4 times the diameter we get 52 where the central cell n is added 4 times and each cell of the circumference only once. So $4 \cdot n = 52 - 40 = 12$. Hence $n = 3$.

 # 16. Masha put a multiplication sign between the 2nd and 3rd digits of the number 2020 and noted that the resulting product $20 \cdot 20$ is a square number. How many numbers between 2010 and 2099 (including 2020) have the same property?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

SOLUTION: Solutions are $20 \cdot 20$, $20 \cdot 45$, $20 \cdot 80$. If the 4 digits of the year are 2, 0, a and b, then we get $20 \cdot \overline{ab} = n^2$ for some positive integer n. Because $20 = 5 \cdot 2^2$, $\overline{ab} = 5 \cdot m^2$ for some positive integer m. As $10 \leq \overline{ab} \leq 99$, $m = 2$, $m = 3$ or $m = 4$ are possible.

 # 17. Two squares of different size are drawn inside an equilateral triangle. One side of one of these squares lies on one of the sides of the triangle, as shown. What is the size of the angle marked by the question mark?



(**A**) 25° (**B**) 30° (**C**) 35° (**D**) 45° (**E**) 50°

SOLUTION: The sum of the angles of the pentagon at the top of the figure is 540°. The known angles of this pentagon are 70°, 60°, 90° and 270°. So the missed angle is $540° - 490° = 50°$.

 # 18. Luca began a 520 km trip by car with 14 litres of fuel in the car tank. His car consumes 1 litre of fuel per 10 km. After driving 55 km, he reads a road sign showing the distances from that point to five petrol stations ahead on the road. These distances are 35 km, 45 km, 55 km, 75 km and 95 km. The capacity of the car's fuel tank is 40 litres and Luca wants to stop just once to fill the tank. How far is the petrol station that he should stop at?

(**A**) 35 km (**B**) 45 km (**C**) 55 km (**D**) 75 km (**E**) 95 km

SOLUTION: When Luca reads the information plate he can still drive 85 km at most with the fuel in his tank and has still 465 km to go. So he can't reach the fifth station. As a full tank takes him at most 400 km he should not stop within the next 65 km. So only the fourth station is possible.

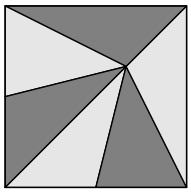
 # 19. Let $17x + 51y = 102$. What is the value of $9x + 27y$?

(**A**) 54 (**B**) 36 (**C**) 34 (**D**) 18

(**E**) The value is undetermined.

SOLUTION: If we divide the given equation by 17 we get $x + 3y = 6$. If we multiply this by 9 we get $9x + 27y = 54$.

 # 20. A square shaped stained glass window of 81 dm^2 is made out of six triangles of equal area (see figure). A fly is sitting exactly on the spot where the six trangles meet. How far from the bottom of the window is the fly sitting?



(**A**) 3 dm (**B**) 5 dm (**C**) 5.5 dm (**D**) 6 dm (**E**) 7.5 dm

SOLUTION: The triangle on the upper side of the window has an area of 1/6 of the whole window. So its height is equal to 1/3 of the height of the window which is 9 dm. So the fly is sitting 6 dm above the bottom.

5 points

 # 21. The digits from 1 to 9 are randomly arranged to make a 9-digit number. What is the probability that the resulting number is divisible by 18?

(**A**) $\frac{1}{2}$ (**B**) $\frac{4}{9}$ (**C**) $\frac{5}{9}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{3}{4}$

SOLUTION: All such numbers are divisible by 9 because their digit sum is 45. So the last digit must be even, hence the probability is $\frac{4}{9}$.

 # 22. A hare and a tortoise competed in a 5 km race along a straight line. The hare is five times faster than the tortoise. The hare mistakenly started perpendicular to the route. After a while he realized his mistake, then turned and ran straight to the finish point. He arrived at the same time as the tortoise. What is the distance between the hare's turning point and the finish point?

(**A**) 11 km (**B**) 12 km (**C**) 13 km (**D**) 14 km (**E**) 15 km

SOLUTION: Denote the strating point S, the turning point T and the finish point F.

Then we have $\|SF\| = 5$ km, $\|TF\| = x$ km and $\|ST\| = 25 - x$ km since arriving at the same time means that the hare covered the distance of 25 km. The Pythogorean theorem implies $5^2 + (25 - x)^2 = x^2$ and we get $x = 13$ km.

 # 23. There are some squares and triangles on the table. Some of them are blue and the rest are red. Some of these figures are large and the rest are small. We know the following two facts are true:

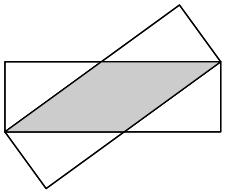
1) if the figure is large then it is a square and
2) if the figure is blue then it is a triangle.

Which of the statements A–E must be true?

(**A**) All red figures are squares. (**B**) All squares are large. (**C**) All small figures are blue.
(**D**) All triangles are blue. (**E**) All blue figures are small.

SOLUTION: There might be red triangles, hence A and D may not be true. There might be small squares, hence B may not be true. There might be small figures that are red, hence C may not be true. E must be true because every blue figure is a triangle, and every large figure is a square, so every blue figure is small.

 # 24. Two identical rectangles with sides of length 3 cm and 9 cm overlap, as shown in the diagram.



What is the area of the overlap of the two rectangles?

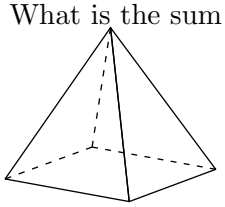
(**A**) 12 cm^2 (**B**) 13.5 cm^2 (**C**) 14 cm^2 (**D**) 15 cm^2 (**E**) 16 cm^2

SOLUTION: Because of the symmetry horizontal unshaded side and longer skew part are equal. We denote the length of these segments by x.

Then the length of the shaded horizontal side is $9-x$. The Pythogorean theorem gets us $3^2+x^2 = (9-x)^2$, so $x = 4$.

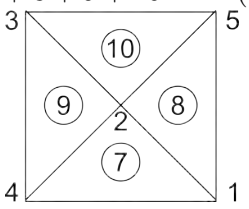
The shaded area is $27 - 3 \cdot 4 = 15$.

 # 25. Kanga labelled the vertices of the square-based pyramid using 1, 2, 3, 4 and 5 once each. For each face Kanga calculated the sum of the numbers on its vertices. Four of these sums are 7, 8, 9 and 10. What is the sum of the numbers at the the vertices of the fifth face?



(**A**) 11 (**B**) 12 (**C**) 13 (**D**) 14 (**E**) 15

SOLUTION: The sum of numbers at the vertices of the base is at least $1+2+3+4 = 10$. If it equals 10 then the top of the pyramid is marked with 5 and all four sums are at least $5+1+2 = 8 > 7$ — a contradiction. Hence the sums 7, 8, 9 and 10 are calculated for the side faces. Therefore if the top vertex is marked with x then $7+8+9+10 = 2 \cdot (1+2+3+4+5) + 2x$. Hence $x = 2$ and the fifth sum equals $1+3+4+5 = 13$.



 # 26. A large cube is built using 64 smaller identical cubes. Three of the faces of the large cube are painted. What is the maximum possible number of small cubes that have exactly one face painted?

(**A**) 27 (**B**) 28 (**C**) 32 (**D**) 34 (**E**) 40

SOLUTION: There are only two possibilities: 3 faces around the corner or U-shape that is two opposite faces and one in between.

In the first case we have 27 small cubes with exactly one face painted (three times 3×3).

In the second case we have 32 small cubes (two times 3×4 plus 2×4).

 # 27. Anna wants to write a number in each of the squares of the grid so that the sum of the four numbers in each row and the sum of the four numbers in each column are the same. Se has already written some numbers, as shown. What number does she write in the shaded square?

1		6	3
	2	2	8
	7		4
		7	

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

SOLUTION: A formal way: denote the number in the bottom right corner by x then the sum must be $x + 15$ and fill out the table.

A nice way: the first line and the second column have an empty cell in common so the cell in the 4th line and the second column must contain 1.

Now the last line and the last column have the bottom right corner in common so we have to have 7 in the bottom left corner.

 # 28. Alice, Belle and Cathy had an arm-wrestling contest. In each game two girls wrestled, while the third rested. After each game, the winner played the next game against the girl who had rested. In total, Alice played 10 times, Belle played 15 times and Cathy played 17 times. Who lost the second game?

(**A**) Alice

(**B**) Belle

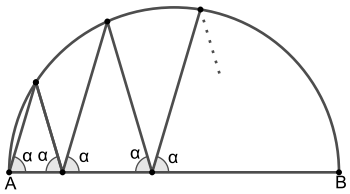
(**C**) Cathy

(**D**) either Alice or Belle could have lost the second game

(**E**) either Belle or Cathy could have lost the second game

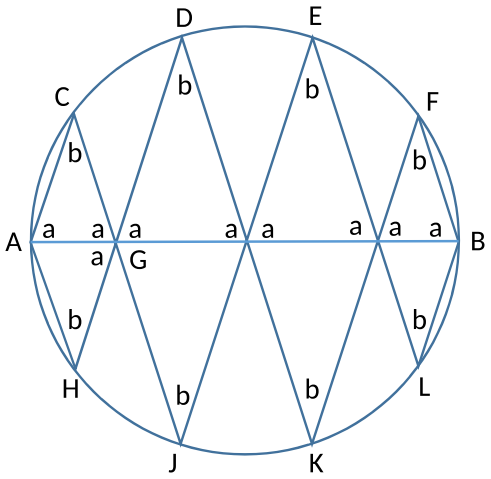
SOLUTION: There were $\frac{10+15+17}{2} = 21$ games. Since Alice was resting during $21 - 10 = 11$ games and nobody gets to rest more than one game in a row, Alice rested during the first, third, fifth, ... games. Therefore, she lost the second game. Such a scenario is possible: First nine games could be BC AB BC AB BC AB BC AB BC (that is, alternating BC AB) and the other 12 alternating AC BC.

 # 29. A zig-zag line starts at the point A, at one end of the diameter AB of a circle. Each of the angles between the zig-zag line and the diameter AB is equal to α as shown. After four peaks, the zig-zag line ends at the point B. What is the size of angle α?



(**A**) 60° (**B**) 72° (**C**) 75° (**D**) 80° (**E**) Another answer

SOLUTION: Reflect the figure along its diametre, so that $arc(AC) = arc(AH), arc(CD) = arc(HJ)$, etc. Note that D,G,H are collinear because of the equality of alternate angles (both a). Also note that AC and GD (and so HD) are parallel. It follows that $arc(CD) = arc(AH)$. In particular, we now have $arc(AC) = arc(AH) = arc(CD) = arc(HJ)$. Repeating the argument we in fact have (by symmetry or parallel lines) that all arcs in the figure are equal, namely $arc(AC) = ... = arc(BL)$. So the decagon $ACDEFBLKJH$ is regular. In particular $a = 144^o/2 = 72^o$. // Other solution: For reasons of symmetry it is clear that the zig-zag line goes through the centre M of the circle. Because they are all isoscele triangles with the same angle α at their base, $\triangle AGC$, $\triangle GMD$ and $\triangle CAM$ are similar to each other and $\triangle GMD$ and $\triangle CAM$ are congruent because they both have the radius of the circle as their longer side. Because of $arc(AC) = arc(CD)$ (see above) the $\triangle DCM$ is congruent too. So two times the angle β is equal to angle α and we get $5 * \beta = 180^o$ or $\alpha = 72^o$.



 # 30. Eight consecutive three-digit positive integers have the following property: each of them is divisible by its last digit. What is the sum of the digits of the smallest of the eight integers?

(**A**) 10 (**B**) 11 (**C**) 12 (**D**) 13 (**E**) 14

SOLUTION: The set of the last digits can only be $1, 2, \ldots, 8$ or $2, 3, \ldots, 8, 9$.

If $\overline{abc}$ is divisible by c, then $\overline{abc} - c$ is divisible by 10 so $\overline{ab0}$ is divisible by c for all c in the sets mentioned in the first line. It follows that $\overline{ab0}$ must be divisible by 2, 3, 5, 7 and 8. So it is divisible by 840. So the smallest number $\overline{abc}$ must be 841.