Year 4

(Entry into Year 5)

25 Hour Revision Course

Mathematics

NORD ANGLIA EDUCATION

Section 1 – Geometry

4 hours

Shape Properties

Any two-dimensional shape made up of straight lines is called a polygon. Although circles and ovals are shapes, they have curved lines, and are therefore not polygons. Polygons can be regular or irregular. Regular polygons are shapes that have equal angles and equal lengths for all sides. Irregular polygons can have different lengths and angles, but the total sum of the angles is the same as for regular polygons.

Some examples of polygons include Triangles, Squares, and Pentagons, although there are others with more sides.

To find the total sum of the interior angles in a polygon, you can do so using the following formula.

$$Total\ Sum\ of\ Angles = (Number\ of\ Sides - 2)\ X\ 180$$

For example, with a Triangle

$$Total\ Sum\ of\ Angles = (3 - 2)\ X\ 180 = 180°$$

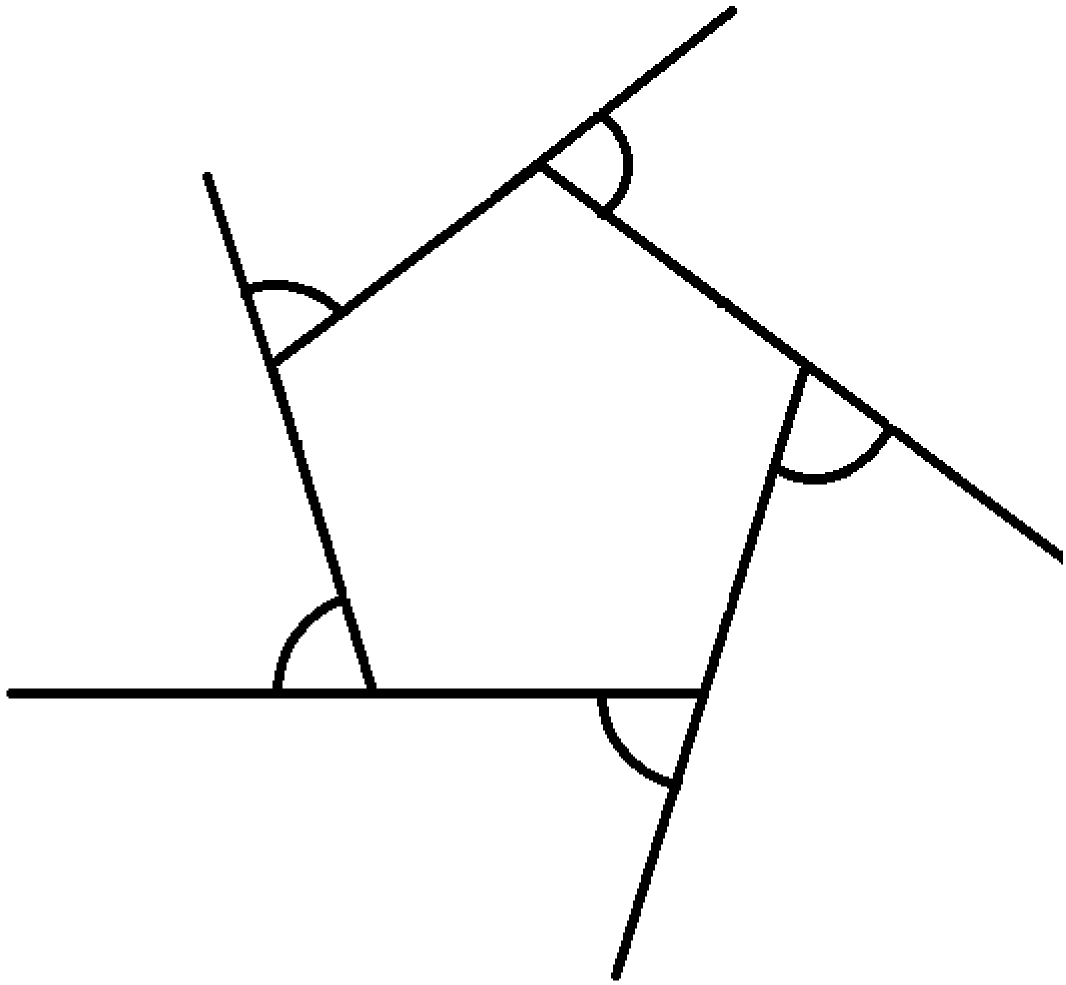
Therefore, the total sum of the interior angles in a Triangle is 180°. For a regular triangle, the size of each angle would be

$$\frac{180°}{Number\ of\ Sides\ (3)} = 60°$$

For exterior angles of polygons, the total sum of the angles is 360°. To find the individual exterior angle of regular polygons, you need to do the following sum;

$$\frac{360°}{Number\ of\ Sides}$$

For the regular 5-sided polygon (Pentagon) below.



$$\frac{360°}{5} = 72°$$

The external angle of a regular Pentagon is 72°.

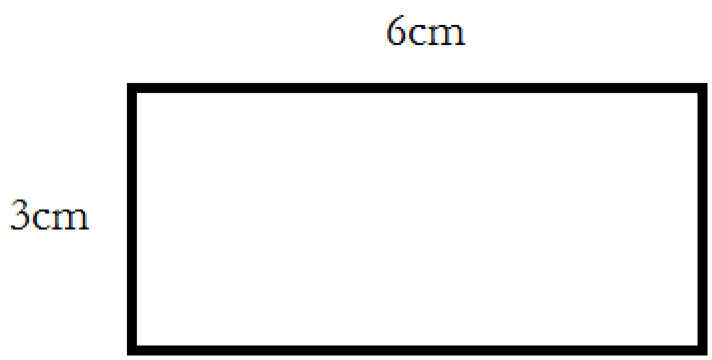
Using the Information above, please fill in the rest of the table below.

Number of Sides	Name of Shape	Total Sum of Angles	Size of Interior angle of a regular polygon	Total Sum of Exterior Angles	Size of Exterior Angle of a regular polygon
3	Triangle	180°	60°	360°	
4				360°	
5				360°	
6				360°	
7				360°	

Perimeter

Perimeter is the distance around a two-dimensional shape, and possibly better described as the length of all of the sides added together. In circles, it is known as the circumference.

For example:



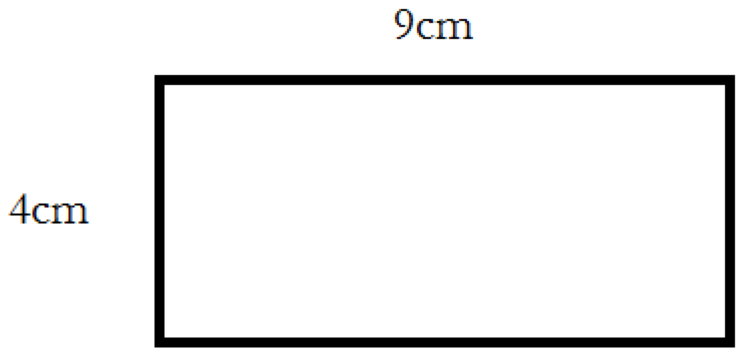
With this rectangle, the length is 6cm and the width is 3cm. In this shape, there are two lengths, and two widths, so the sum we are trying to work out is;

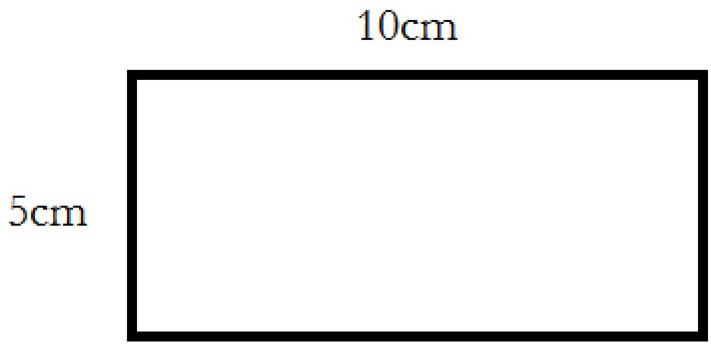
$$(2\ X\ 6) + (2\ X\ 3) = 12 + 6 = 18$$

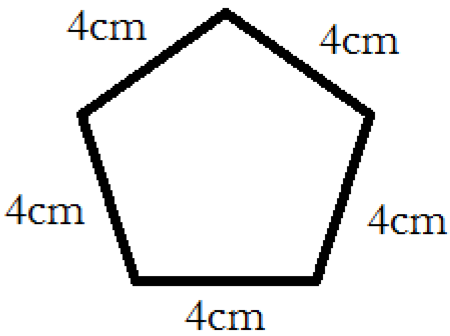
For each of the shapes below, please find the perimeter.

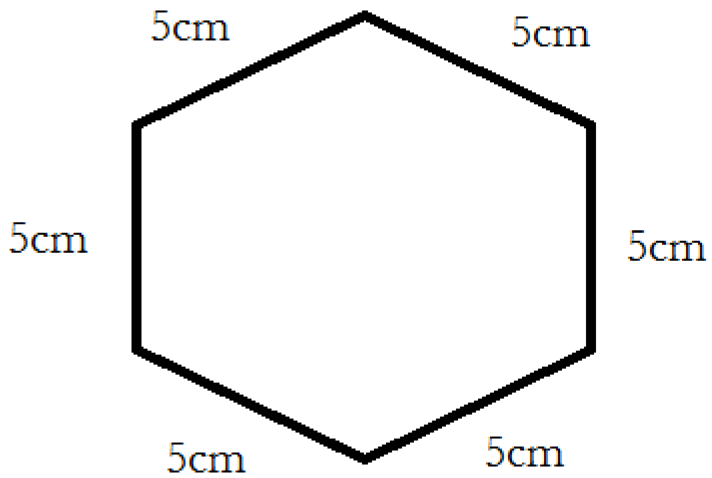
For each of the shapes below, please find the perimeter. Also write the name of the shape next to the shape.

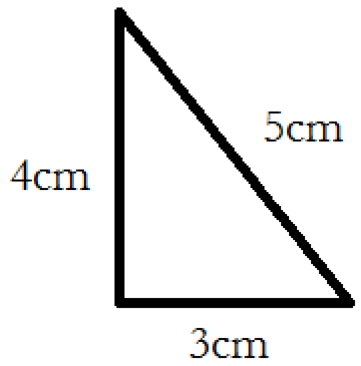
(Drawings are not to scale.)


9cm
4cm


10cm
5cm


4cm
4cm
4cm
4cm
4cm

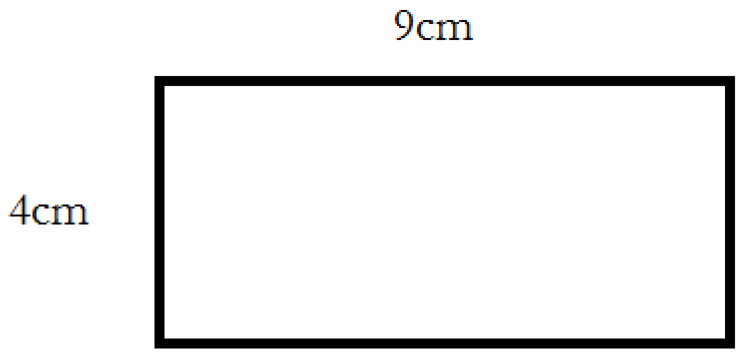

5cm
5cm
5cm
5cm
5cm
5cm


5cm
4cm
3cm

Area

To work out the area of squares and rectangles (4-sided polygons), we multiply the length and the width together.

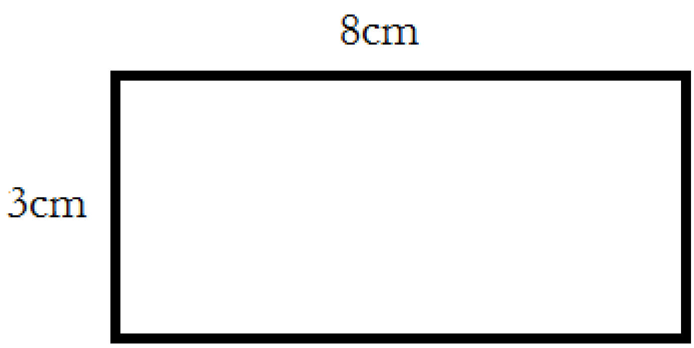
For example;

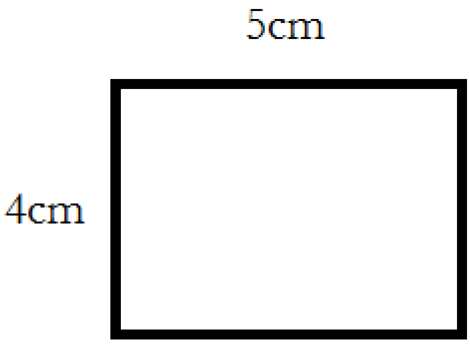


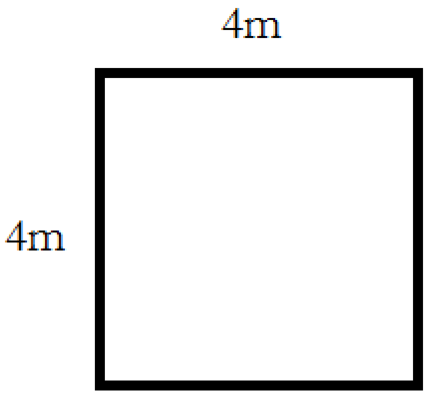
$$Area = Length \times Width = 9\ X\ 4 = 36cm^2$$

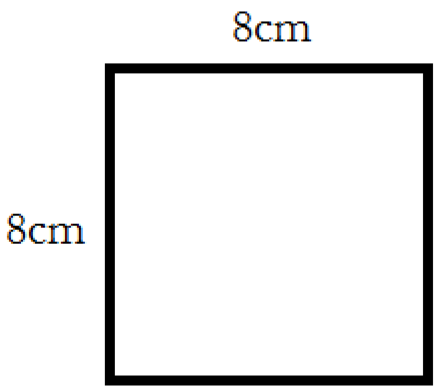
When working out areas, you need to pay attention to the units that you put after your answer. The unit above is square centimetres (written as cm^2). If the lengths of the sides are metres, then the area would be in square metres (m^2)

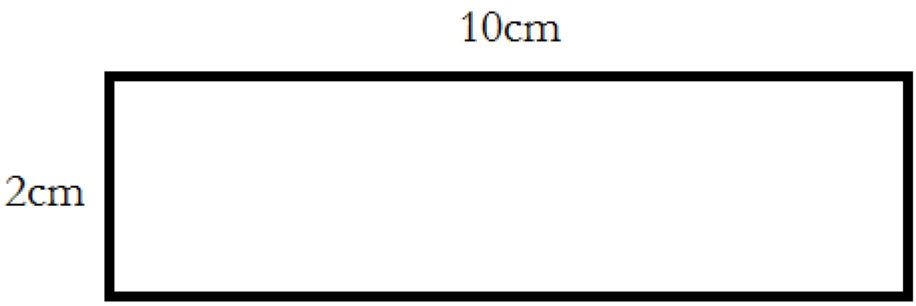
Work out the area of the quadrilaterals below. Some come with a diagram, others will just give the dimensions.




5cm
4cm


4m
4m


8cm
8cm


10cm
2cm

5m × 3m

10cm × 9cm

4cm × 8cm

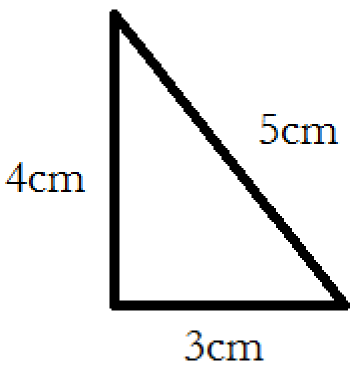
6cm × 6cm

7m × 7m

To work out the area of triangles (3-sided polygons), we multiply the base and the height together, and then we divide by 2. The formula looks like this;

$$Area\ of\ Triangles = \frac{1}{2} \times Base \times Height$$

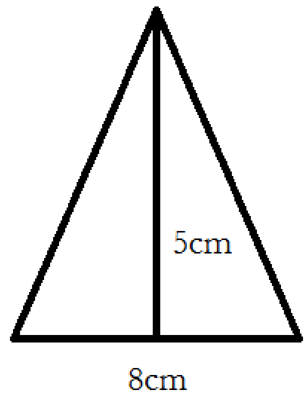
For example;

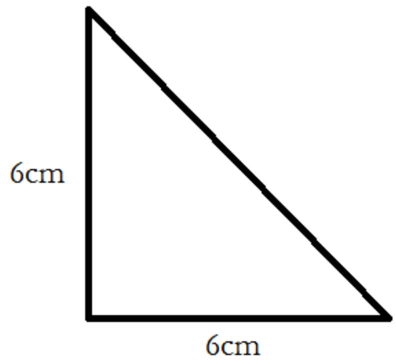


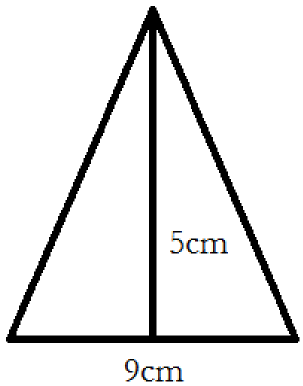
In this example, the base of the triangle is 3cm, and the height is 4cm. Therefore, the area of the Triangle will be as follows;

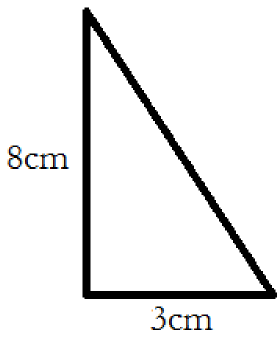
$$Area\ of\ Triangles = \frac{1}{2} \times 3cm \times 4cm = \frac{1}{2} \times 12 = 6cm^2$$

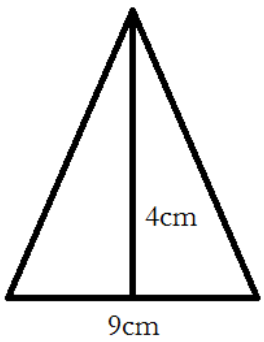
Work out the area of the Triangles below. Some come with a diagram, others will just give the dimensions.











Base: 5cm	Height: 6cm
Base: 2cm	Height: 12cm
Base: 5cm	Height: 4cm
Base: 8cm	Height: 8cm
Base: 10cm	Height: 5cm

Section 2 – Order of Operations

2 hours

Order of Operations

'Operations' are the parts of sums that indicate the actions taking place in the sum. Examples of operations include adding, subtracting, multiplying, or dividing. Sometimes we are presented with a sum that is confusing in its layout. For example;

$$4 + 3 \times 2 + 4$$

Fortunately, there is an order that we can use to help us to solve this problem.

B Brackets

O Order

D Division

M Multiplication

A Addition

S Subtraction

By using the BODMAS order, we can make sure that we solve all difficult questions correctly. The DMAS are areas that we have already covered, but Brackets and Order may require further explanation. Brackets are operations or numbers within brackets, like this (6+2). These are always done first. Order, or Indices as they are sometimes known, refer to powers of numbers, such as x^2 or x^3.

Here are some worked examples of BODMAS at work.

$4 + 3 \times 2$

$= 4 + 6$

$= 10$

$5 \times (3-1) \div 2$

$= (5 \times 4) \div 2$

$= 10$

$3 \times 2^2 - 3$

$= (3 \times 4) – 3$

$= 9$

Work out the answer to the sums below. Please pay attention to the Order of Operations.

$20 \div 4 + 17 \times 6$

$14 \div 7 - 5 \times 5$

$20 \div 4 - 3 + 6$

$7 - 4 \times 12 \div 2$

$12 \div 6 \times 2 + 13$

$8 \div 4 \times 6 + 2$

$10 \div 2 + 6 \times 12$

$8 + 8 - 24 \div 12$

$17 - 11 \times 18 \div 2$

$4 + 7 \times 11 - 3$

$(42 - 2) \div 8 - 3^2$

$(4 \times 7 - 5^2) - 4$

$(34 - 6) \div 7 - 2^2$

$3 \times (12 - 3) + 5^2$

$(8 \times 9 + 3^2) - 7$

Section 3 – Multiplication

1 hours

Multiplication

Work out the answer to the Long Multiplication sums below.

	Th	H	T	U
			9	3
	×		1	2
+				

	Th	H	T	U
			8	5
	×		1	5
+				

	Th	H	T	U
			5	4
	×		2	9
+				

	Th	H	T	U
			7	3
	×		1	8
+				

	Th	H	T	U
			4	1
	×		3	3
+				

	Th	H	T	U
		1	4	4
	×		2	7
+				

	Th	H	T	U
		2	2	3
	×		2	0
+				

	Th	H	T	U
		4	1	0
	×		4	2
+				

	Th	H	T	U
		5	8	3
	×		6	5
+				

	Th	H	T	U
		8	6	3
	×		4	8
+				

Section 4 – Division

1 hours

Division

Please work out the following sums using Division (whether Long Division or otherwise).

$6\overline{)\,5\ 4}$

$4\overline{)\,5\ 2}$

$3\overline{)\,7\ 8}$

$5\overline{)\,7\ 7}$

$6\overline{)\,9\ 3}$

$8\overline{)\,1\ 0\ 6}$

$2\ 8\overline{)\,8\ 4\ 0}$

$$36\overline{)684}$$

$$20\overline{)980}$$

$$28\overline{)784}$$

Section 5 – Negative Numbers

2 hours

Negative Numbers

In some questions, we will often be presented with negative numbers to deal with. Here are some tips to remember.

Addition

8 + 3 = 11

8 + -3 = 5

Subtraction

8 - 3 = 5

8 – (-3) = 8

Multiplication

8 × 3 = 24

8 × (-3) = -24

(-8) × 3 = -24

(-8) × (-3) = 24

Division

8 ÷ 4 = 2

8 ÷ (-4) = -2

(-8) ÷ 4 = -2

(-8) ÷ (-4) = 2

Work out the answers to the questions involving negative numbers below.

$8 + (-6) =$

$-6 + 9 =$

$10 + (-7) =$

$6 + (-5) =$

$-2 + 5 =$

$11 - 4 =$

$9 - (-3) =$

$5 - (-4) =$

$12 - (-6) =$

$-8 - 7 =$

$8 \times (-3) =$

$(-12) \times (-2) =$

$5 \times (-7) =$

$(-9) \times (-4) =$

$6 \times (-3) =$

$9 \div (-3) =$

$(-16) \div 4 =$

$(-20) \div (-5) =$

$21 \div (-7) =$

$(-24) \div (-8) =$

Section 6 – Fractions

7 hours

Mixed Numbers

Mixed Numbers are fractions that contain both integers (whole numbers) and fractions. They look something like this;

$$3\frac{1}{2}$$

Mixed numbers often come from Improper fractions (also known as top-heavy fractions). Learning how to convert improper fractions into mixed numbers is extremely important.

<u>To convert Mixed numbers into Improper fractions</u>

In order to convert mixed numbers into improper fractions, we multiply the integer by the denominator of the fraction. Once we have done that, and received our answer, we add in the numerator of the fraction to get our improper numerator.

For example;

$$3\frac{1}{2} = \frac{(3 \times 2) + 1}{2} = \frac{7}{2}$$

<u>To convert Improper fractions into Mixed numbers</u>

To convert improper fractions into mixed numbers, we divide the top-heavy numerator by the denominator of the fraction. This gives us the integer value, and the remainder is the numerator value for the fraction section. The denominator of the fraction is as before.

For example;

$$\frac{9}{4} = 9 \div 4 = 2\,r1$$

$$= 2\,\frac{1}{4}$$

Convert these Mixed numbers into Improper fractions.

$4\frac{1}{2}$

$2\frac{1}{3}$

$2\frac{1}{5}$

$5\frac{3}{4}$

$6\frac{2}{3}$

$10\frac{1}{2}$

$5\frac{3}{7}$

$4\frac{5}{6}$

$3\frac{7}{8}$

$1\frac{7}{12}$

Convert these Improper fractions into Mixed Numbers.

$$\frac{13}{4}$$

$$\frac{16}{5}$$

$$\frac{7}{3}$$

$$\frac{9}{5}$$

$$\frac{16}{6}$$

$$\frac{15}{9}$$

$$\frac{65}{8}$$

$$\frac{44}{7}$$

$$\frac{27}{11}$$

$$\frac{25}{12}$$

Adding and Subtracting Mixed Numbers

To add and subtract mixed numbers, first you must convert the numbers into improper fractions. Once you have done that, you can add or subtract the fractions as normal. If the denominators are the same, you simply add or subtract the numerators. If the denominators are different, then you need to make them the same by finding the Lowest Common Multiple and finding the equivalent fractions for each part of the equation. Once the denominators are the same, add or subtract as necessary. Your answer may be an improper fraction; if it is, then follow the steps required to turn it into a mixed number. You should not give answers as improper fractions, and answers should be in their simplest form, where possible.

For example;

$$2\frac{1}{5} + 3\frac{1}{3} = \frac{(2 \times 5) + 1}{5} + \frac{(3 \times 3) + 1}{3} = \frac{11}{5} + \frac{10}{3} = \frac{33}{15} + \frac{50}{15} = \frac{83}{15} = 5\frac{8}{15}$$

Please solve these additions using Mixed Numbers.

$4\frac{1}{2} + 6\frac{2}{5} =$

$3\frac{1}{2} + 7\frac{2}{3} =$

$4\frac{1}{2} + 6\frac{2}{5} =$

$6\frac{1}{2} + 4\frac{2}{5} =$

$2\frac{5}{10} + 3\frac{3}{5} =$

$3\frac{1}{4} + 4\frac{1}{3} =$

$1\frac{1}{2} + 8\frac{9}{10} =$

$3\frac{3}{2} + 5\frac{1}{3} =$

$6\frac{2}{3} + 4\frac{2}{6} =$

$5\frac{1}{5} + 8\frac{6}{10} =$

Please solve these subtractions using Mixed Numbers.

$9\frac{1}{2} - 1\frac{2}{10} =$

$6\frac{8}{10} - 3\frac{2}{3} =$

$8\frac{2}{4} - 3\frac{1}{10} =$

$6\frac{3}{4} - 3\frac{5}{8} =$

$8\frac{6}{10} - 1\frac{1}{5} =$

$7\frac{8}{10} - 3\frac{2}{3} =$

$9\frac{4}{5} - 1\frac{12}{15} =$

$8\frac{5}{12} - 3\frac{3}{4} =$

$5\frac{3}{5} - 1\frac{1}{10} =$

$7\frac{3}{4} - 2\frac{1}{3} =$

Multiplying and Dividing Mixed Numbers

To multiply and divide mixed numbers, first you must convert them to improper fractions. Once this is done, you multiply or divide as you would a normal fraction, cross-cancelling if you are able to. To finish, turn the answer from an improper fraction to a mixed number (if applicable).

For example;

$$2\frac{1}{5} \times 3\frac{1}{3} = \frac{(2 \times 5) + 1}{5} X \frac{(3 \times 3) + 1}{3} = \frac{11}{\cancel{5}1} \times \frac{\cancel{10}2}{3} = \frac{22}{3} = 7\frac{1}{3}$$

Please solve these multiplications using Mixed Numbers.

$2\frac{1}{2} \times 3\frac{9}{10} =$

$2\frac{2}{3} \times 4\frac{4}{5} =$

$2\frac{1}{2} \times 3\frac{2}{3} =$

$3\frac{1}{5} \times 4\frac{1}{2} =$

$2\frac{9}{10} \times 3\frac{4}{5} =$

$4\frac{1}{2} \times 4\frac{1}{3} =$

$3\frac{2}{5} \times 2\frac{1}{3} =$

$4\frac{1}{4} \times 3\frac{1}{3} =$

$3\frac{1}{3} \times 3\frac{2}{5} =$

$4\frac{1}{2} \times 3\frac{2}{3} =$

Please solve these divisions using Mixed Numbers.

$2\frac{1}{5} \div 2\frac{1}{4} =$

$3\frac{3}{5} \div 2\frac{2}{3} =$

$4\frac{1}{2} \div 2\frac{1}{5} =$

$2\frac{3}{4} \div 3\frac{4}{5} =$

$5\frac{3}{8} \div 1\frac{3}{4} =$

$5\frac{1}{2} \div 3\frac{1}{5} =$

$3\frac{4}{6} \div 3\frac{4}{6} =$

$3\frac{1}{3} \div 2\frac{9}{10} =$

$5\frac{3}{5} \div 2\frac{1}{5} =$

$3\frac{2}{3} \div 3\frac{4}{5} =$

Section 7 – Factors

2 hours

Prime Factorization

Factorization is finding which numbers multiply together to get a number. Prime Factorization is about finding which Prime numbers multiply together to make a number. It is particularly useful to do, as it have help us to find the Highest Common Factor.

To work out the Prime Factors of a number, you divide it by the lowest possible prime numbers repeatedly, until you reach 1. When you reach 1, make a note of the factors, and write them as below. For example, for the number 42;

$$\begin{array}{r|r} 2 & 42 \\ 3 & 21 \\ 7 & 7 \\ & 1 \end{array}$$

The prime factors of 42 are **$2 \times 3 \times 7$**.

Another example, 60;

$$\begin{array}{r|r} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

The prime factors of 60 are **$2^2 \times 3 \times 5$**.

List the Prime Factors of each of the following numbers

4

12

40

65

72

80

95

110

38

56

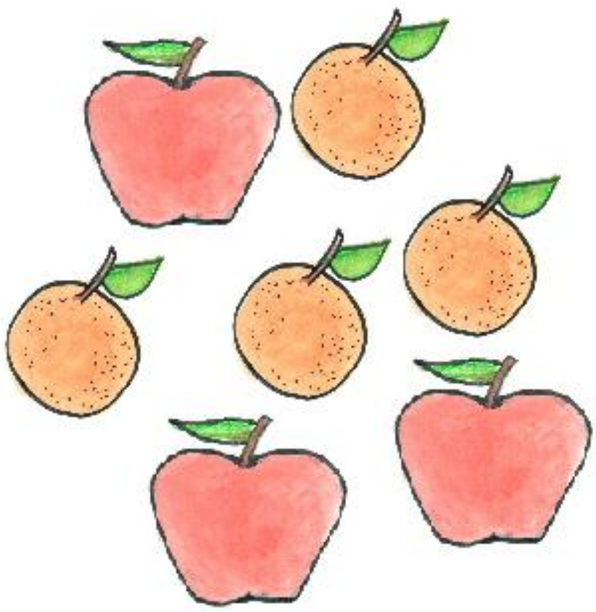
Section 8 – Ratios

3 hours

Ratios

Ratios explain how much of there is of one thing compared to another thing. They are useful in everyday life as they can help with things such as getting the right proportions for recipes, but they are also extensively used in Maths, for things such as working out equivalent fractions of for separating objects into groups according to a prescribed proportion.

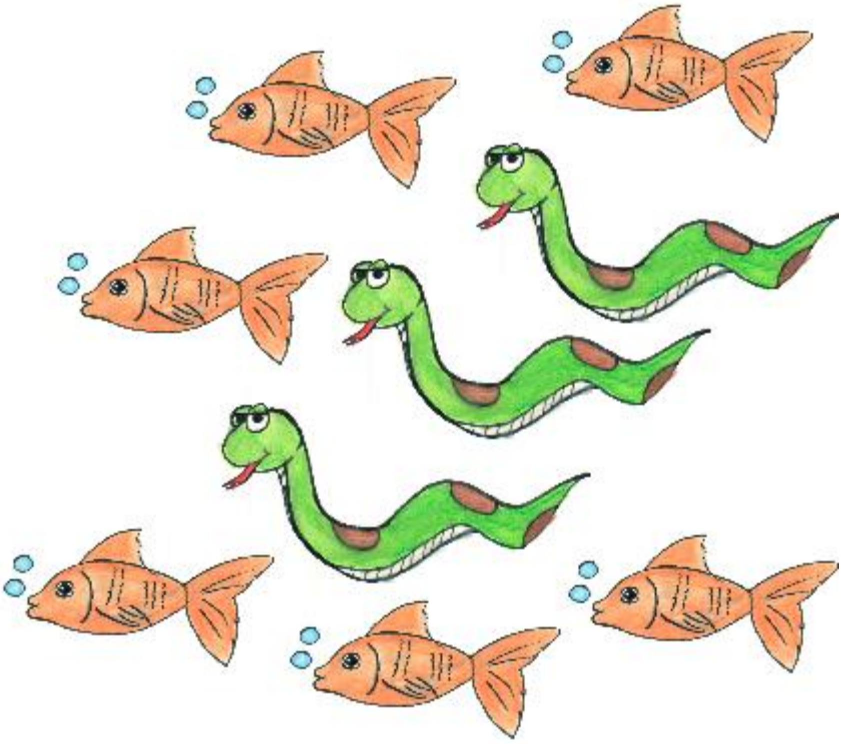
For example;

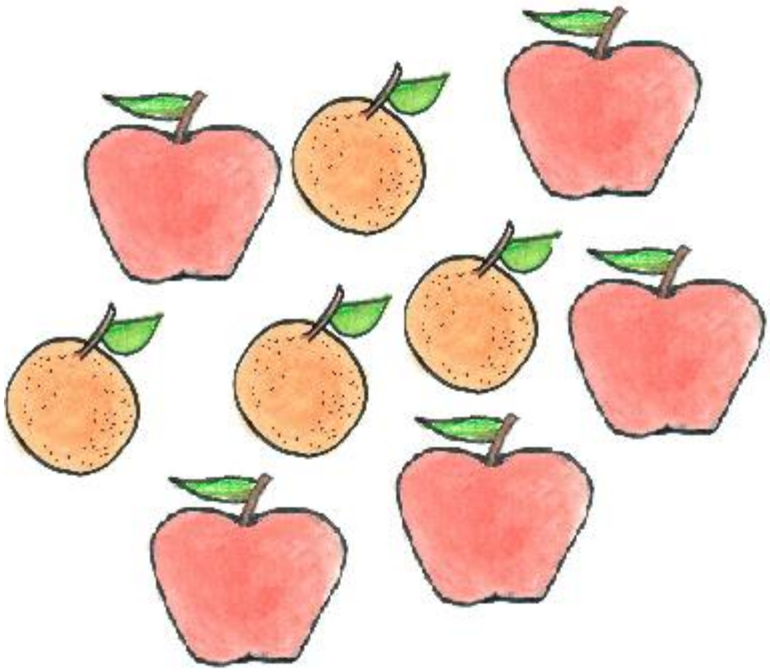


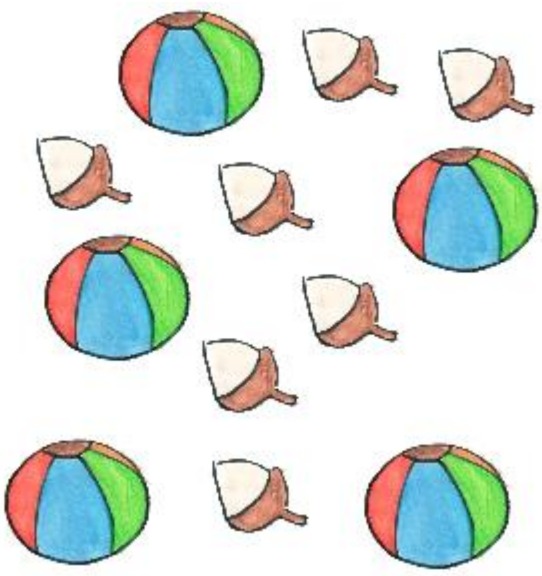
What is the ratio of Oranges to Apples?

There are 4 Oranges and 3 Apples, so the ratio is 4:3.

What are the ratios of the objects below?







To find out whether the following fractions are equivalent, divide one fraction by another. The numerator and denominator should be the same number of times larger or smaller.

For example;

$\frac{2}{5}$ *and* $\frac{10}{25}$ are equivalent. 10 ÷ 2 is 5, and 25 ÷ 5 is also 5.

$\frac{3}{4}$ *and* $\frac{16}{20}$ are not equivalent. 16 ÷ 3 is 5.33333, and 20 ÷ 4 is 5. These numbers are different, so they are not equivalent.

Work out whether the following fractions are equivalent.

$\frac{2}{4}$ *and* $\frac{12}{20}$

$\frac{3}{4}$ *and* $\frac{15}{20}$

$\frac{5}{6}$ *and* $\frac{30}{36}$

$\frac{1}{2}$ *and* $\frac{8}{14}$

$\frac{1}{2}$ *and* $\frac{25}{50}$

$\frac{1}{4}$ *and* $\frac{3}{12}$

$\frac{1}{4}$ *and* $\frac{15}{50}$

$\frac{7}{8}$ *and* $\frac{40}{35}$

$\frac{4}{7}$ *and* $\frac{20}{35}$

$\frac{9}{10}$ *and* $\frac{36}{45}$

Using your knowledge of equivalent ratios, find the unknown value

$$\frac{5}{14} = \frac{a}{42}$$

$$\frac{3}{4} = \frac{12}{b}$$

$$\frac{c}{2} = \frac{6}{12}$$

$$\frac{5}{d} = \frac{20}{24}$$

$$\frac{3}{5} = \frac{e}{30}$$

$$\frac{9}{11} = \frac{36}{f}$$

$$\frac{g}{7} = \frac{42}{49}$$

$$\frac{63}{h} = \frac{9}{10}$$

$$\frac{4}{7} = \frac{i}{21}$$

$$\frac{3}{7} = \frac{9}{j}$$

Ratio Problems using words

Sometimes we are not always given pictorial evidence to help us build ratios, sometimes we only have words. When tackling the examples below, please put them in their simplest form. To do this, divide both sides of the ratio by the Highest Common Factor (HCF)

Using your knowledge of ratios, identify the ratio in its simplest form

12 beetles out of 66 insects ____________________

4 points out of 28 points ____________________

10 cheeses to 25 cheeses ____________________

6 inches to 12 inches ____________________

28 cakes out of 56 cakes ____________________

15 footballs to 18 footballs ____________________

18 rainy days out of 54 days ____________________

4 feet out of 14 feet ____________________

30 snowy days out of 33 days ____________________

42 metres to 77 metres ____________________

4 miles out of 20 miles ____________________

54 blue cars out of 72 cars ____________________

21 red bikes out of 84 bikes ____________________

8 swans out of 22 swans ____________________

18 pints to 20 pints ____________________

35 pounds to 60 pounds ____________________

15 kilos to 21 kilos ____________________

4 litres to 40 litres ____________________

42 cups to 56 cups ____________________

20 hours to 50 hours ____________________

Section 9 – Probability

2 hours

Probability

Probability in Maths involves estimating how likely (or probable) it is that something will happen. Probability can be used to predict a range of daily events, from simple actions such as tossing a coin, to whether it's likely to snow today.

Estimating Probability

Probability is often used in daily life. In our conversations, we often evaluate the probability of events happening.

e.g. Is it going to rain today?

- No, it's unlikely…

As estimates, we can use 7 basic statements to evaluate probability;

- Certain
- Very Likely
- Likely
- Even
- Unlikely
- Very Unlikely
- Impossible

Using these statements, evaluate the probability of the following points;

- That it will rain today
- That more people speak English than Swahili
- That pigs can fly
- That I will eat dinner today

As will become apparent, there are some questions which allow for subjective interpretation. Each person will have their own view and assessment of the probability. In other cases, there is no doubt – the answer is fixed.

Which of these points is subjective?

- That car is good
- That car is white

Basic Probability

Although estimates for probability are useful, their subjectivity makes life tricky. If different people have different views, which causes problems (see the rain example above…)

Mathematicians, and people in general, prefer certainty. Certain events can be accurately mapped and are definite. Their probability ranges from 0 (impossible) to 1 (certain).

Probability can be expressed as a fraction, decimal, or percentage

Events of Equal Probability

Events of equal probability refer to areas where each value should be as likely as another. Examples of this involve tossing a coin, or rolling a dice

$$Probability\ of\ an\ outcome = \frac{Number\ of\ ways\ an\ outcome\ can\ happen}{Total\ Number\ of\ possible\ outcomes}$$

For example

$$Probability\ of\ rolling\ a\ 2 = \frac{1\ (There's\ only\ one\ 2\ on\ a\ dice)}{6\ (6\ sides\ on\ a\ dice}$$

$$Probability\ of\ rollling\ an\ even\ number = \frac{3\ (2,4,6)}{6} = \frac{1}{2}$$

Please answer the following questions on probability

Fill in the boxes to show the probabilities of different actions when flipping a coin

	Probability
Heads	
Tails	

Fill in the boxes to show the probabilities of different actions when rolling a dice

	Probability
1	
2	
3	
4	
5	
6	

- What is the probability of rolling an odd number?
- What is the probability of rolling a 2 or a 4?
- What is the probability of rolling 1, 3, 5, or 6?

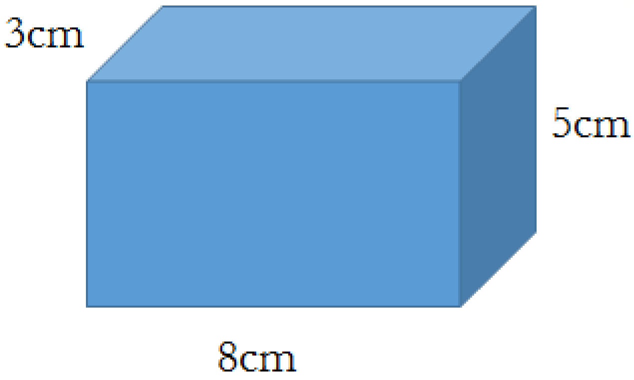
Section 10 – Volume

1 hours

Volume

Earlier in the booklet, we explained Perimeter and Area. These are measurements used within two-dimensional shapes. Volume helps us to measure the amount of space that a substance or object occupies in the third-dimension.

For cuboids, volume is identified as length × width × height. For example;



In this shape, the volume would be **8cm × 5cm × 3cm = 120cm^3**. Please note that the unit is cubed

Work out the volume of the following cuboids

Length: 4cm	Width: 3cm	Height: 5cm

Length: 6cm	Width: 2cm	Height: 5cm

Length: 3cm	Width: 7cm	Height: 4cm

Length: 2cm	Width: 5cm	Height: 3cm

Length: 4cm	Width: 4cm	Height: 4cm

Length: 6cm	Width: 6cm	Height: 6cm

Length: 10cm	Width: 5cm	Height: 8cm

Length: 1cm	Width: 3cm	Height: 6cm

Length: 2.5cm	Width: 5cm	Height: 7cm

Length: 9cm	Width: 2cm	Height: 5cm