3 points

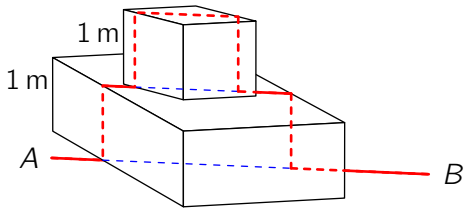
 # 1. What is the sum of the last two digits of the product $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$?

请问乘积$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$结果的后面两位数字之和是多少?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 16

 # 2. An ant walked every day on a straight horizontal path from A to B, which are 5 m apart. One day, humans placed on its path two strange obstacles of height 1 m each. The ant still walks along the same straight line except that now it has to climb up and down vertically over the boxes, as shown below. How long is its path now?

一只蚂蚁每天从A点直线行走至B点。该路程的距离为5m。有一天，一个人在蚂蚁走的这条路上放置了两个高度均为1m的盒子以作为障碍物。这只蚂蚁依旧沿着相同的直线行走，不过它现在必须在两个障碍物上垂直向上和向下爬行。下图显示了蚂蚁的爬行路线，请问这只蚂蚁现在需要爬行的距离是多远?

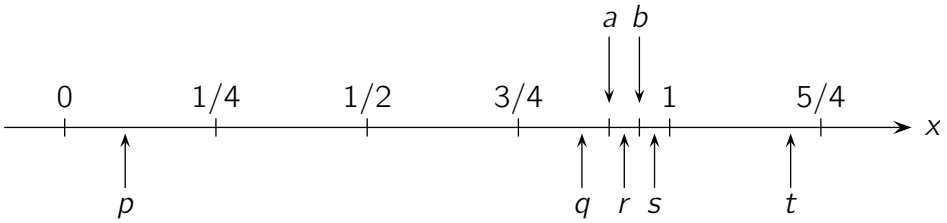


(**A**) 7 m (**B**) 9 m (**C**) $5+4\sqrt{2}$ m (**D**) $9-2\sqrt{2}$ m (**E**) 10 m

---

 # 3. Rene marked two points a and b on the number line. Which point represents the product ab?

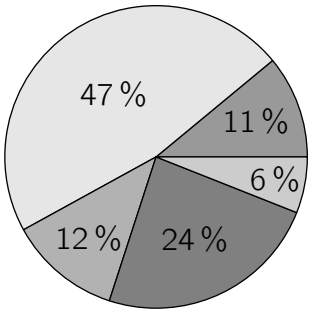
Rene在数轴上标记了两个点，a点和b点。请问数字线上的p、q、r、s和t点，哪一个最有可能代表着ab乘积?



(**A**) p (**B**) q (**C**) r (**D**) s (**E**) t

 # 4. The pie chart shows how students travel to get to a school. Approximately twice as many travel by bicycle as those who use public transport. Roughly the same number travel by car as those who walk to school. The rest use a motorcycle. What percentage of the students use a motorcycle?

以下的圆形统计图显示学生去学校的方式。骑自行车的学生人数大约是使用公共交通工具的两倍，坐私家车的学生人数则与步行的学生人数非常相近。其余的便是骑摩托车去上学的学生。请问骑摩托车去学校的百分比是多少?



(**A**) 6% (**B**) 11% (**C**) 12% (**D**) 24% (**E**) 47%

 # 5. The sum of five three-digit numbers is 2664, as shown below. What is the value of $A+B+C+D+E$?

以下的算式显示的是5个三位数的总和为2664。请问A + B + C + D + E的值是多少?

$$\begin{array}{r} A\,B\,C \\ +\,B\,C\,D \\ +\,C\,D\,E \\ +\,D\,E\,A \\ +\,E\,A\,B \\ \hline 2\,6\,6\,4 \end{array}$$

(**A**) 4 (**B**) 14 (**C**) 24 (**D**) 34 (**E**) 44

 # 6. What is the value of $\dfrac{1010^2+2020^2+3030^2}{2020}$?

$\dfrac{1010^2+2020^2+3030^2}{2020}$ 的值是多少?

(**A**) 2020 (**B**) 3030 (**C**) 4040 (**D**) 6060 (**E**) 7070

 # 7. Let a, b and c be integers satisfying $1 \leqslant a \leqslant b \leqslant c$ and $abc = 1\,000\,000$. What is the largest possible value of b?

设a, b, 和c都是整数且满足1 ⩽ a ⩽ b ⩽ c 和abc = 1 000 000。请问b可能的最大值是多少?

(**A**) 100 (**B**) 250 (**C**) 500 (**D**) 1000 (**E**) 2000

 # 8. If D dogs weigh K kilograms and E elephants weigh the same as M dogs, how many kilograms does one elephant weigh?

如果D只狗的体重是K千克，而E只大象的体重和M只狗的体重是一样的，那么一头大象的体重是多少千克

(**A**) $DKEM$ (**B**) $\frac{DK}{EM}$ (**C**) $\frac{KE}{DM}$ (**D**) $\frac{KM}{DE}$
(**E**) $\frac{DM}{KE}$

 # 9. There are two dice. Each one has two red faces, two blue faces and two white faces. If we roll both dice together, what is the probability that both dice show the same color?

有两个骰子，每个骰子都有两面红色、两面蓝色和两面白色。如果我们掷这两个骰子，那么这两个骰子都显示相同颜色的概率是多少?

(**A**) $\frac{1}{12}$ (**B**) $\frac{1}{9}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{2}{9}$ (**E**) $\frac{1}{3}$

 # 10. Which of the following numbers is not divisible by 3 for any integer n?

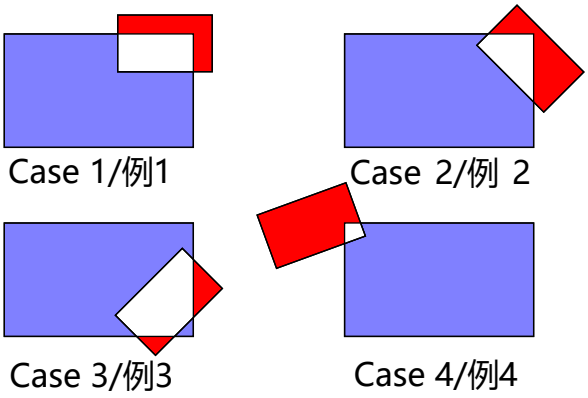
下列哪一项，对于任意整数n都不能被3整除?

(**A**) $5n+1$ (**B**) n^2 (**C**) $n(n+1)$ (**D**) $6n-1$ (**E**) n^3-2

4 points

 # 11. A blue rectangle and a red rectangle are overlapping. The figure shows 4 different such cases. We denote by B the area of the part of the blue rectangle that is not common to the two rectangles, and we denote by R the area of the red rectangle that is not common to the two. Which of the following statements is true about the quantity $B - R$?

一个蓝色的矩形和一个红色的矩形有部分重叠。下图中显示了4种不同的情况，我们用B表示蓝色矩形中除两个矩形重叠部分外的面积，用R表示红色矩形中除两个矩形重叠部分外的面积。关于B - R，下列哪个陈述是正确的?



(**A**) In case 1 the quantity B-R is larger than in the other cases
在例1中，B-R的面积比其他例子都大

(**B**) In case 2 the quantity B-R is larger than in the other cases
在例2中，B-R的面积比其他例子都大

(**C**) In case 3 the quantity B-R is larger than in the other cases
在例3中，B-R的面积比其他例子都大

(**D**) In case 4 the quantity B-R is larger than in the other cases
在例4中，B-R的面积比其他例子都大

(**E**) The quantity B-R is the same in all cases
所有例子中的B-R的面积都相同

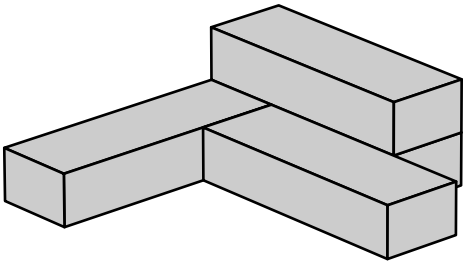
 # 12. Five coins are lying on a table with the "heads" side up. At each step, you must turn over exactly three of the coins. What is the least number of steps required to have all the coins with the "tails" side up?

将五枚硬币以正面朝上的方式摆放在桌子上，每一次必须翻转三枚硬币。请问最少需要多少次才能将所有硬币的反面朝上。

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

 # 13. Four identical boxes are glued together to make the structure shown below. One liter of paint is needed to paint the outside of one such box. How many liters of paint are needed to paint the outside of the glued structure?

四个相同的盒子粘在一起以形成如图所示的形状。一个盒子的外部需要一公升的油漆来上色。如果要将图中盒子的外部全部涂上油漆，请问需要多少公升的油漆?



(**A**) 2.5 (**B**) 3 (**C**) 3.25 (**D**) 3.5 (**E**) 4

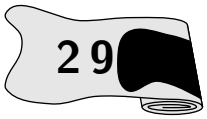
 # 14. Let a, b and c be integers. Which of the following is certainly NOT equal to $(a-b)^2+(b-c)^2+(c-a)^2$?

设 a，b，c为整数。请问 $(a-b)^2+(b-c)^2+(c-a)^2$肯定不等于下列哪一项?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 6 (**E**) 8

 # 15. The first two digits of a 100-digit integer are 2 and 9. How many digits does the square of this number have?

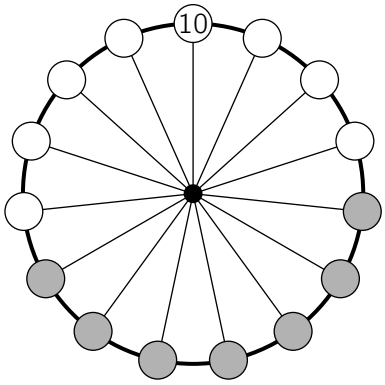
有一个100位的整数，其首两个数字是2和9。请问这个100位整数的平方会有多少位数?



(**A**) 101 (**B**) 199 (**C**) 200 (**D**) 201
(**E**) Cannot be determined / 无法确定

 # 16. Matjaz has placed 15 numbers on a wheel. Only one of the numbers is visible, the 10 at the top. The sum of the numbers in any 7 consecutive positions on the wheel, such as the ones shaded grey, is always the same. When all 15 numbers are added, exactly how many of the numbers 75, 216, 365 and 2020 are possible totals?

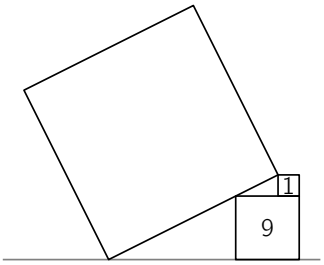
Matjaz在一个轮子中输入15个数字，其中只有一个数字可见，即顶部的10。在任何7个连续位置上的圆（如灰色的圆）中的数字总和总是相同的。当所有15个数字都加起来时，可能得到75、216、365和2020中的几个数?



(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

 # 17. A large square touches two other squares, as shown in the diagram. The numbers in the small squares represent their areas. What is the area of the large square?

如图所示，一个大正方形触碰到另外两个小正方形。小正方形中的数字代表其面积。请问大正方形的面积是多少?



(**A**) 49 (**B**) 80 (**C**) 81 (**D**) 82 (**E**) 100

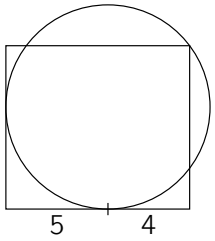
 # 18. The sequence f_n is given by $f_1 = 1$, $f_2 = 3$ and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 1$. Among the first 2020 elements of the sequence, how many are even?

序列f_n给出$f_1 = 1$, $f_2 = 3$和$f_{n+2} = f_n + f_{n+1}$, 其中$n \geq 1$。请问在序列的前2020个元素中，有多少是偶数?

(**A**) 673 (**B**) 674 (**C**) 1010 (**D**) 1011 (**E**) 1347

 # 19. A circle and a rectangle are drawn in such a way that the circle touches two of the sides of the rectangle and passes through one of its vertices. What is the area of the rectangle?

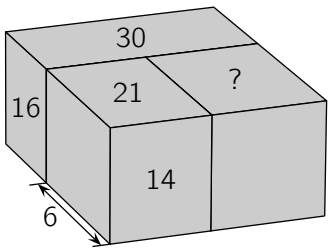
下图显示一个圆形和一个矩形。圆形与矩形的其中两边相切并且穿过其中一个顶点。请问该矩形的面积是多少?



(**A**) 27π (**B**) 25π (**C**) 72 (**D**) 63 (**E**) 81

 # 20. Three cuboids are arranged to make a larger cuboid. The width of one of them is 6 and the areas of some of their faces are 14, 21, 16, 30, as shown below. What is the area of the face with the question mark?

以下的长方体是由三个不同大小的长方体所组成的。如图显示, 其中一个长方体的宽度为6，其中一些长方体表面的面积为14，21，16，30。请问图中带问号的面积是多少?

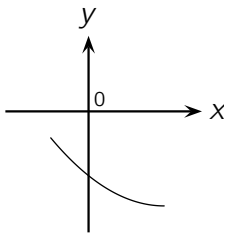


(**A**) 18 (**B**) 24 (**C**) 28 (**D**) 30 (**E**) 32

5 points

 # 21. The figure shows a section of the parabola with equation $y = ax^2 + bx + c$. Which of the following numbers must be positive?

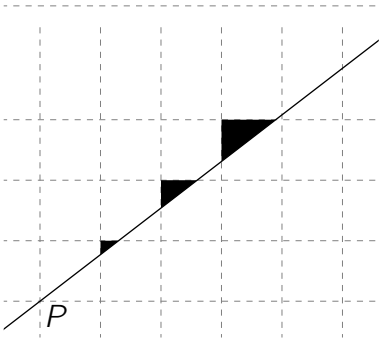
下图显示了抛物线的一部分，其方程式为y = ax² + bx + c。请问下列哪个选项一定是正数?



(**A**) c (**B**) $b + c$ (**C**) ac (**D**) bc (**E**) ab

 # 22. On a square grid paper, a line passes through the lower left corner P. Which of the following could be the ratio of the areas of the three shaded triangles?

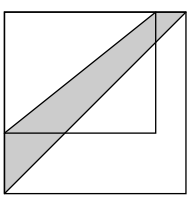
在一张方格纸上，有一条线穿过左下角P。请问下列哪个是三个阴影三角形的面积比例?



(**A**) 1 : 2 : 3 (**B**) 1 : 2 : 4 (**C**) 1 : 3 : 9 (**D**) 1 : 4 : 8 (**E**) 1 : 4 : 9

 # 23. The length of one of the sides of a rectangular garden is increased by 20% and the length of the other side is increased by 50%. The new garden is a square, as shown in the diagram. The shaded area between the diagonal of the square garden and the diagonal of the original rectangular garden is 30 m^2. What was the area of the original rectangular garden?

下图为一个矩形花园，其某一边的长度增加了20%，另一边的长度增加了50%，然后产生的这个新花园是一个正方形，如图所示。正方形花园对角线与原矩形花园对角线之间的阴影面积为30 m^2。请问原来矩形花园的面积是多少?



(**A**) 60 m^2 (**B**) 65 m^2 (**C**) 70 m^2 (**D**) 75 m^2 (**E**) 80 m^2

 # 24. An integer N is divisible by all integers from 2 to 11, except two of them. Which pair of integers could be the exceptions?

存在一个整数N，可被2到11中的所有整数整除，但不能被其中两个整数整除，请问可能是以下哪两个整数?

(**A**) 2 & 3 (**B**) 4 & 5 (**C**) 6 & 7 (**D**) 7 & 8 (**E**) 10 & 11

 # 25. In the morning, an ice-cream shop offers 16 flavors. Anna wants to choose a 2–flavor ice cream. In the evening, several flavors are sold out. Bella wants to choose a 3–flavor ice cream from the remaining flavors. Both Anna and Bella can choose from the same number of possible combinations. How many flavors were sold out?

在早上的时候，一家冰淇淋店提供了16种口味的冰淇淋。Anna想选其中2种口味的冰淇淋。到了傍晚，有几种口味的冰淇淋都已经卖完了，此时Bella想要从剩下的口味中选出3种口味的冰淇淋。然而Anna和Bella可以选择的口味的组合数量却是相同的。请问在那个时候有多少种口味的冰淇淋已经被卖完?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

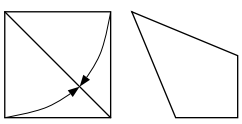
 # 26. Tony has 71 marbles in a box. He is allowed to take out exactly 30 marbles from the box or to return exactly 18 marbles into it. Tony is allowed to apply each operation as many times as he wishes. What is the smallest number of marbles than could be in the box?

Tony的盒子里有71颗弹珠。他可以从盒子里拿出30颗弹珠，或归还18颗弹珠。Tony可以按自己的意愿多次执行这个操作。那么，盒子里的弹珠数量最少可以是多少?

(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

 # 27. Wajda took a square piece of paper of side 1 and folded two of its sides to the diagonal, as shown in the diagram, to make a quadrilateral. What is the area of this quadrilateral?

Wajda拿了一张边长为1的正方形纸，把它的两条边对折到对角线上，如图所示，形成一个四边形，请问这个四边形的面积是多少?



(**A**) $2-\sqrt{2}$ (**B**) $\frac{\sqrt{2}}{2}$ (**C**) $\sqrt{2}-1$ (**D**) $\frac{7}{10}$ (**E**) $\frac{3}{5}$

 # 28. An iceberg has the shape of a cube. Exactly 90% of its volume is hidden below the surface of the water. Three edges of the cube are partially visible over the water. The visible parts of these edges have lengths 24 m, 25 m and 27 m. How long is an edge of the cube?

有一座冰山的形状是正方体。其体积的90%正好隐藏在水面以下。正方体的其中三条边有些部分可在水面上看得见。这些看得见的边的部分的长度分别为24m，25m和27m。正方体的实际边长度是多少?

(**A**) 30 m (**B**) 33 m (**C**) 34 m (**D**) 35 m (**E**) 39 m

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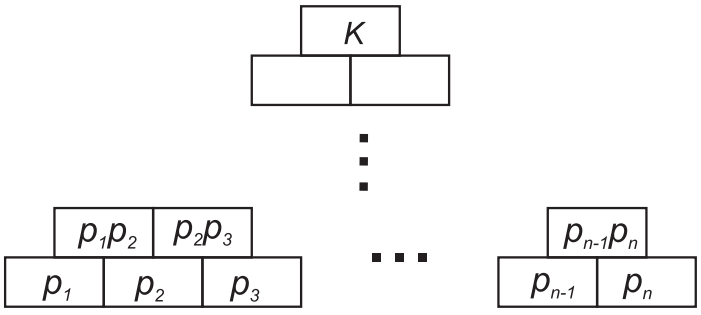
 # 29. There are n different prime numbers p_1 to p_n written from left to right in the bottom row of the table shown. The product of two numbers next to each other in the same row is written in the box directly above them. The number $K = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_n^{\alpha_n}$ is written in the box in top row. In a table where $\alpha_2 = 8$, how many numbers are divisible by the number p_4?

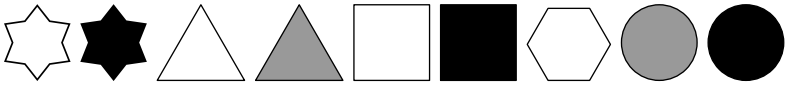
在下图所示的三角形格子的底行中，从左到右有n个不同的质数p_1到p_n。将同一行中彼此相邻的两个数字的乘积写在它们正上方的格子中。数$K = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_n^{\alpha_n}$写在最顶端的格子中。如果$\alpha_2 = 8$，请问有多少个格子当中的数字可被$p_4$整除?



(**A**) 4 (**B**) 16 (**C**) 24 (**D**) 28 (**E**) 36

 # 30. Adam and Britt try to find out which of the following figures is Carl's favourite.

Adam和Britt想要找出以下哪个图形是Carl最喜欢的图形。



Adam knows that Carl has told Britt its shape. Britt knows that Carl has told Adam its colour. Then the following conversation takes place. Adam: "I don't know Carl's favourite figure and I know that Britt doesn't know it either." Britt: "At first I didn't know Carl's favourite figure, but now I do." Adam: "Now I know it too." Which figure is Carl's favourite?

Adam知道Carl已经告诉了Britt关于这个图形的形状，Britt知道Carl已经告诉Adam关于这个图形的颜色。然后发生了接下来的对话，Adam: "我不知道Carl最喜欢的图形，我知道Britt也不知道。Britt: "起初我不知道Carl最喜欢的图形是什么，但现在我知道了。" Adam: "现在我也知道了。" 请问Carl最喜欢哪个图形?

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

END OF PAPER

-Scratch Paper-

草稿纸

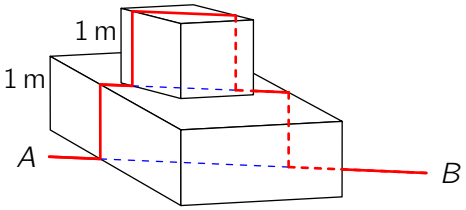
3 points

 # 1. What is the sum of the last two digits of the product $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 16

SOLUTION:

 # 2. An ant walked every day on a straight horizontal line path from A to B, which are 5 m apart. One day humans placed on its path two strange obstacles of height 1 m each. Now the ant walks along or above the same straight line except that it now has to climb up and down vertically over both the two obstacles, as in the picture. How long is its path now?

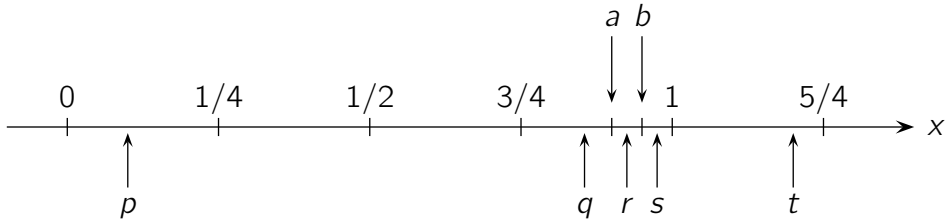


(**A**) 7 m (**B**) 9 m (**C**) $5 + 4\sqrt{2}$ m (**D**) $9 - 2\sqrt{2}$ m

(**E**) the length depends on the angles the obstacles are situated along the path

SOLUTION: The horizontal part of the path is exactly as long as the original path AB. The can be seen by projecting the new path onto the horizontal plane. The extra length comes from the four vertical parts of the path. The total length is 5+(1+1+1+1)=9 m.

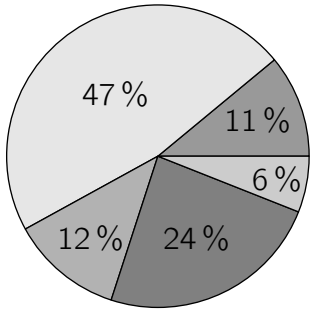
 # 3. Rene marked two points a and b as accurately as possible on the number line. Which of the points p, q, r, s, t on the number line best represents their product ab?



(**A**) p (**B**) q (**C**) r (**D**) s (**E**) t

SOLUTION: Both a and b are less than 1 so their product is less than 1. In fact $ab < a$ which excludes points r, s, t. Also certainly $a, b > \frac{1}{2}$ so $ab > \frac{1}{4}$so certainly p is excluded. This leaves q which, moreover, is realistically placed because $b \approx 1$ so $ab \approx a$, so ab is close but less that a. Such is the case as with the location of q.

 # 4. The pie chart shows how the students of my school get to school. Approximately twice as many go by bike as use public transport and roughly the same number come by car as walk. The rest use a moped. What percentage use a moped?



(**A**) 6 % (**B**) 11 % (**C**) 12 % (**D**) 24 % (**E**) 47 %

SOLUTION:

 # 5. The sum of five three-digit numbers is 2664, as shown on the board. What is the value of $A+B+C+D+E$?

$$\begin{array}{r} \mathbf{A\,B\,C} \\ +\,\mathbf{B\,C\,D} \\ +\,\mathbf{C\,D\,E} \\ +\,\mathbf{D\,E\,A} \\ +\,\mathbf{E\,A\,B} \\ \hline \mathbf{2\,6\,6\,4} \end{array}$$

(**A**) 4 (**B**) 14 (**C**) 24 (**D**) 34 (**E**) 44

SOLUTION: $(100A+10B+C)+(100B+10C+D)+(100C+10D+E)+(100D+10E+A)+(100E+10A+B) = 111(A+B+C+D) = 2664$ so $A+B+C+D = 2664 : 111 = 24$.

 # 6. What is the value of $\dfrac{1010^2+2020^2+3030^2}{2020}$?

(**A**) 2020 (**B**) 3030 (**C**) 4040 (**D**) 6060 (**E**) 7070

SOLUTION: $\dfrac{1010^2+2^2\cdot 1010^2+3^2\cdot 1010^2}{2\cdot 1010} = \dfrac{(1+2^2+3^2)1010^2}{2\cdot 1010}$.

 # 7. Let a, b and c be integers satisfying $1 \leqslant a \leqslant b \leqslant c$ and $abc = 1\,000\,000$. What is the largest possible value of b?

(**A**) 100 (**B**) 250 (**C**) 500 (**D**) 1000 (**E**) 2000

SOLUTION:

 # 8. If D dogs weigh K kilograms and E elephants weigh the same as M dogs, how many kilograms does one elephant weigh?

(**A**) $DKEM$ (**B**) $\frac{DK}{EM}$ (**C**) $\frac{KE}{DM}$ (**D**) $\frac{KM}{DE}$ (**E**) $\frac{DM}{KE}$

SOLUTION:

 # 9. There are two dice. Each one has two red faces, two blue faces and two white faces. If we roll both dice together, what is the probability that both show the same color?

(**A**) $\frac{1}{12}$ (**B**) $\frac{1}{9}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{2}{9}$ (**E**) $\frac{1}{3}$

SOLUTION:

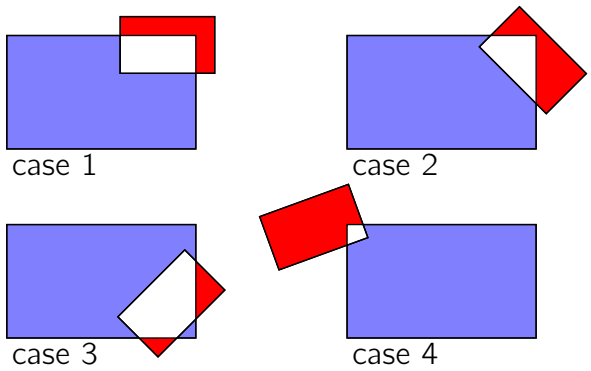
 # 10. Which of the following numbers is not divisible by 3 for any integer n?

(**A**) $5n+1$ (**B**) n^2 (**C**) $n(n+1)$ (**D**) $6n-1$ (**E**) n^3-2

SOLUTION:

4 points

 # 11. A blue rectangle and a red rectangle are overlapping. The figure shows 4 different such cases. We denote by B the area of the part of the blue rectangle that is not common to the two rectangles, and we denote by R the area of the red rectangle that is not common to the two. Which of the following statements is true about the quantity $B-R$?



(**A**) In case 1 the quantity B-R is larger than in the other cases

(**B**) In case 2 the quantity B-R is larger than in the other cases

(**C**) In case 3 the quantity B-R is larger than in the other cases

(**D**) In case 4 the quantity B-R is larger than in the other cases

(**E**) The quantity B-R is the same in all cases

SOLUTION: If W denotes the common area then in each case B-R= (B+W)-(R+W)= (area of blue rectangle)-(area of red rectangle) = constant.

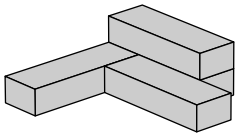
 # 12. Five coins are lying on a table with the "heads" side up. At each step you must turn over exactly three of the coins. What is the least number of steps required to have all the coins lying with the "tails" side up?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5

(**E**) It's not possible to have all the coins with their "tails" side up.

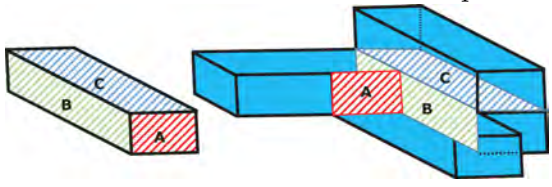
SOLUTION:

 # 13. Four identical boxes are glued together to make the shape shown in the picture. One litre of paint is needed to paint the outside of one such box. How many litres of paint are needed to paint the outside of the glued construction?



(**A**) 2.5 (**B**) 3 (**C**) 3.25 (**D**) 3.5 (**E**) 4

SOLUTION: We would need 4 litres of paint but because of overlaps (where two boxes meet), we need to subtract the areas that are covered. They are of three types. a) The two red areas A, b) the two green areas B and c) the two blue areas C. However, observing the box on the left, two A's, two B's and two C's make the entire box. In short the common parts amount to an entire box, so we need 4-1 = 3 litres of paint.



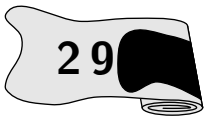
 # 14. Let a, b and c be integers. Which of the following is certainly NOT equal to $(a-b)^2+(b-c)^2+(c-a)^2$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 6 (**E**) 8

SOLUTION: One is not possible since if one of the squares is non-zero, at least one another is as well. Others are possible:

$$\begin{cases} (0-0)^2+(0-0)^2+(0-0)^2 = 0+0+0 = 0, \\ (1-0)^2+(0-1)^2+(1-1)^2 = 1+1+0 = 2, \\ (1-2)^2+(2-3)^2+(3-1)^2 = 1+1+4 = 6, \quad \text{and} \\ (2-0)^2+(0-2)^2+(2-2)^2 = 4+4+0 = 8. \end{cases}$$

 # 15. The first two digits of a 100-digit integer are 2 and 9. How many digits does the square of this number have?

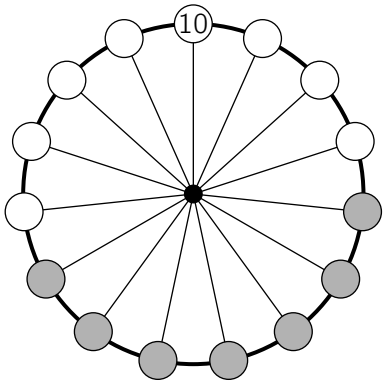


(**A**) 101 (**B**) 199 (**C**) 200 (**D**) 201

(**E**) It cannot be determined

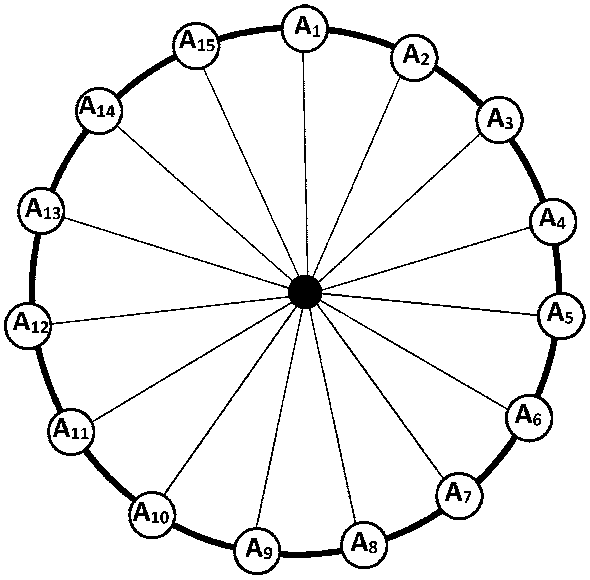
SOLUTION: $10^{99} < A < 30 \times 10^{98}$, so $10^{198} < A^2 < 900 \times 10^{196} < 1000 \times 10^{196} = 10^{199}$. It follows that A^2 has 199 digits.

 # 16. Matjaz has placed 15 numbers on a wheel. Only one of the numbers is visible, the 10 at the top. The sum of the numbers in any 7 consecutive positions on the wheel, such as the ones shaded grey, is always the same. When all 15 numbers are added, exactly how many of the numbers 75, 216, 365 and 2020 are possible totals?

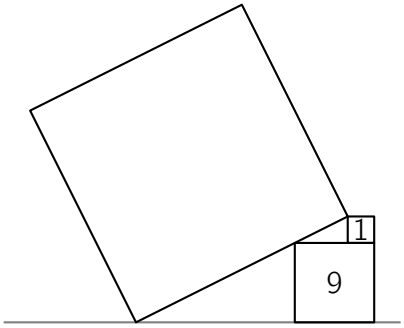


(**A**) zero (**B**) one (**C**) two (**D**) three (**E**) four

SOLUTION: Call the numbers A_n $n = 1$ to 15, with $A_1 = 10$. Since $A_1 + \ldots + A_7 = A_2 + \ldots + A_8$, it follows that $A_1 = A_8$. Similarly $A_8 = A_{15}$, so in fact $A_1 = A_8 = A_{15}$. If we continue around the wheel we will find that $A_1 = A_8 = A_{15} = A_7 = A_{14} = \ldots$ etc. As a matter of fact in this list of equal numbers you will find all fifteen of $A_1, \ldots, A_{15}$. This is fairly obvious since A_{15} is the immediate predecessor of A_1, so the next equal number (here A_7) is the immediate predecessor of a number in the list. So few turns around the wheel in jumps of 7 will pass from all the numbers on the wheel. In other words, all the numbers in the list are equal to each other, namely equal to 10. So the sum of all the numbers in the list is $15 \times 10 = 150$, no other possibility arising. So the answer is "none".



 # 17. A large square touches two other squares, as shown in the diagram. The numbers in the small squares represent their areas. What is the area of the large square?



(**A**) 49 (**B**) 80 (**C**) 81 (**D**) 82 (**E**) 100

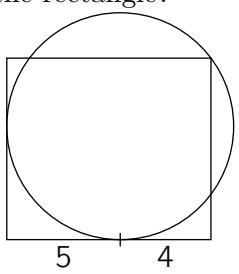
Solution:

 # 18. The sequence f_n is given by $f_1 = 1$, $f_2 = 3$ and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 1$. How many of the first 2020 elements of the sequence are even?

(**A**) 673 (**B**) 674 (**C**) 1010 (**D**) 1011 (**E**) 1347

Solution: We look at parity. The first few terms are $1, 1, 0, 1, 1, 0, 1, 1, ...$ Note that the forth and fifth terms are a repetition of the first two and it is clear that the sequence is periodic. Namely, it has the three terms $1, 1, 0$ repeating eternally, with a zero (even number) every third term. As $2020 = 673 \times 3 + 1$ we have in total 673 zero's.

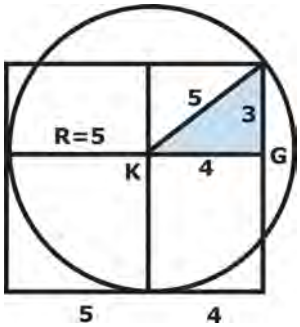
 # 19. A circle and a rectangle have been drawn in such a way that the circle touches two of the sides of the rectangle and passes through one of its vertices. The distances of two vertices of the rectangle from one of the points where the circle touches the rectangle are 5 and 4, as shown. What is the area of the rectangle?



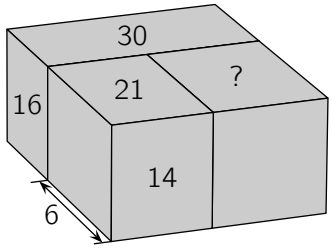
(**A**) 27π (**B**) 25π (**C**) 72 (**D**) 63

(**E**) none of the previous

SOLUTION: Drawing the perpendiculars to the sides of rectangle at the touching points E and F, we note that they pass from the centre K of the circle. It follows that the radius R = KF = AE = 5. So from Pythagoras on KGC we get GC=3 and so the vertical side of the rectangle has length AD=3+5=8. The required area is $8 \times 9 = 72$.



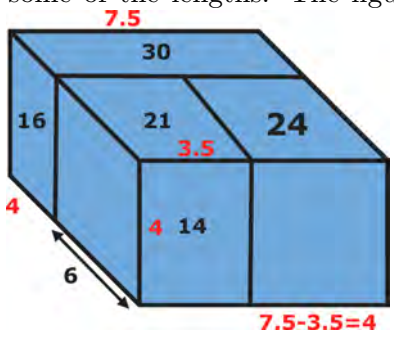
 # 20. Three cuboids are arranged to make a larger cuboid as in the figure. The width of one of them is 6 and the areas of some of their faces are 14, 21, 16, 30, as shown. What is the area of the face with the question mark?



(**A**) 18 (**B**) 24 (**C**) 28 (**D**) 30

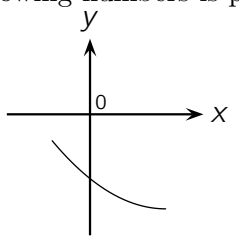
(**E**) cannot be determined

SOLUTION: From the face of area 21 and width 6, we conclude that its length is 3.5. Using this it is easy to determine some of the lengths. The figure describes the results. So the required area is $(7.5 - 3.5) \times 6 = 24$.



5 points

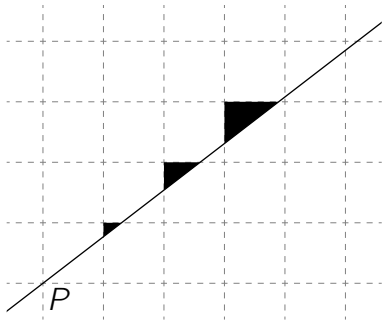
 # 21. The figure shows a section of the parabola with equation $y = ax^2 + bx + c$. Which of the following numbers is positive?



(**A**) c (**B**) $b+c$ (**C**) ac (**D**) bc (**E**) ab

SOLUTION:

 # 22. On a square grid paper, a little kangaroo draws a line passing through the lower left corner P of the grid and colours in three triangles as shown.



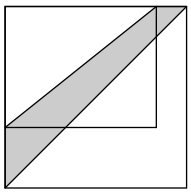
Which of the following could be the ratio of the areas of the triangles?

(**A**) 1 : 2 : 3 (**B**) 1 : 2 : 4 (**C**) 1 : 3 : 9 (**D**) 1 : 4 : 8

(**E**) None of the previous is correct

SOLUTION: Similar triangles with side ratios 1:2:3

 # 23. The length of one of the sides of a rectangular garden is increased by 20% and the length of the other side is increased by 50%. The new garden is a square, as shown in the diagram. The shaded area between the diagonal of the square garden and the diagonal of the original rectangular garden is $30\,\mathrm{m}^2$. What was the area of the original rectangular garden?



(**A**) $60\,\mathrm{m}^2$ (**B**) $65\,\mathrm{m}^2$ (**C**) $70\,\mathrm{m}^2$ (**D**) $75\,\mathrm{m}^2$ (**E**) $80\,\mathrm{m}^2$

SOLUTION: Call $6x$ the side of the square, so then the rectangle has sides $5x$ and $4x$. The shaded area can be divided in two triangles of areas $(x \cdot 4x)/2$ and $(2x \cdot 6x)/2$.

 # 24. A large integer N is divisible by all except two of the integers from 2 to 11. Which of the following pairs of integers could be these exceptions?

(**A**) 2 and 3 (**B**) 4 and 5 (**C**) 6 and 7 (**D**) 7 and 8 (**E**) 10 and 11

SOLUTION:

 # 25. In the morning, the ice-cream shop offers 16 flavours. Anna wants to choose a 2-flavour ice cream. In the evening several flavours are sold out and Bella wants to choose a 3-flavour ice cream from those flavours left. Both Anna and Bella can choose from the same number of possible combinations. How many flavours were sold out?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

SOLUTION:

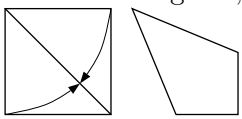
 # 26. Tony has 71 marbles at his disposal in a box. He is allowed to take out exactly 30 marbles from the box or to return exactly 18 marbles to it. Tony is allowed to apply each operation as many

times as he wishes. What is the smallest number of marbles than can be in the box?

(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

SOLUTION: $(30, 18) = 6$ and the biggest number less than 71 and divisible by 6 is 66. Thus, $71-66 = 5$ is the least possible number of marbles that could remain in the box. One possible realization is $71 - 30 - 30 + 18 + 18 + 18 - 30 - 30 = 5$.

 # 27. Wajda took a square piece of paper of side 1 and folded two of its sides to the diagonal, as shown in the diagram, to make a quadrilateral. What is the area of this quadrilateral?



(**A**) $2 - \sqrt{2}$ (**B**) $\frac{\sqrt{2}}{2}$ (**C**) $\sqrt{2} - 1$ (**D**) $\frac{7}{10}$ (**E**) $\frac{3}{5}$

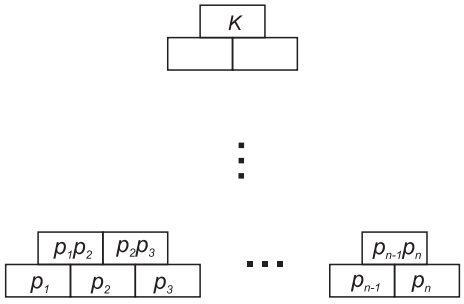
SOLUTION:

 # 28. An iceberg has the shape of a cube. Exactly 90% of its volume is hidden below the surface of the water. Three edges of the cube are partially visible over the water. The visible parts of these edges are 24m, 25m and 27m. How long is an edge of the cube?

(**A**) 30 m (**B**) 33 m (**C**) 34 m (**D**) 35 m (**E**) 39 m

SOLUTION:

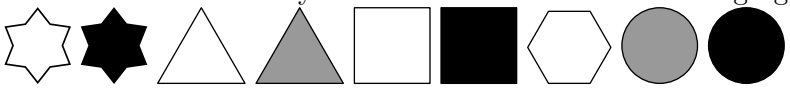
 # 29. There are n different prime numbers p_1 to p_n written from left to right in the bottom row of the table shown. The product of two numbers next to each other in the same row is written in the box directly above them. The number $K = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}$ is written in the box in top row. In a table where $\alpha_2 = 8$, how many numbers are divisible by the number p_4?



(**A**) 4 (**B**) 16 (**C**) 24 (**D**) 28 (**E**) 36

SOLUTION:

 # 30. Adam and Britt try to find out which of the following figures is Carl's favourite.



Adam knows that Carl has told Britt its shape. Britt knows that Carl has told Adam its colour. Then the following conversation takes place. Adam: "I don't know Carl's favourite figure and I know that Britt doesn't know it either." Britt: "At first I didn't know Carl's favourite figure, but now I do." Adam: "Now I know it too." Which figure is Carl's favourite?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

SOLUTION: