



# 2019 Hong Kong Mathematics Kangaroo Contest

## — Student —

### 2019香港數學袋鼠競賽

### 【中學高年級】

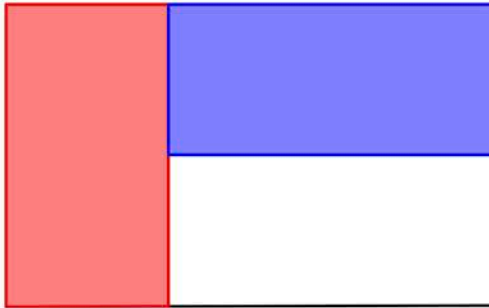
#### Instruction 說明

1. DO NOT FLIP OPEN THIS FRONT COVER UNTIL YOUR PROCTOR TELLS YOU.  
在未收到監考老師指示前，請不要翻開此封面。
2. This is a 30 question multiple choice test. For each question, only one answer choice is correct.  
這是一套包括30道選擇題的測試，每道題目只有一個正確答案。
3. Each question is given a point value. You will receive full points for correct answer, and zero point for blank or incorrect answer. The full score of this test is 120 points.  
每道題目都有給定的分值，答對得滿分，答錯或空白得0分。本次測試的滿分為120分。
4. Mark your answer to each problem on the answer form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be scored.  
請將每道題目的答案用#2鉛筆標註在答題卡上。請注意檢查塗寫的黑色圓圈的準確性，用橡皮完全擦掉錯誤的答案和多餘的標記。只有恰當標註在答題卡上的答案才會被評分。
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.  
只能使用草稿紙、方格紙、尺、量角器和橡皮作為輔助工具。計算器是不允許使用的。測試中沒有任何問題必須需要使用計算器。
6. Figures are not necessarily drawn to scale.  
圖形不一定按比例繪製。
7. Before beginning the test, make sure to record your name, school name and Competition ID on the answer form, especially to bubble in the 8-digit Competition ID completely!  
在開始測試之前，請確保已將你的名字，校名和准考證號填寫在答題卡上，特別是8位准考證號的每位數字已經塗好相應的黑色圓圈。
8. You will have 75 minutes to complete the test once your proctor tells you to begin.  
監考老師宣布開始後，你將有75分鐘的時間完成測試。

**Part 1: 10 problems, 3 points each | 第一部分：10道題目，每題3分**

1. The flag of Kangoroland is a rectangle which is divided into three smaller equal rectangles as shown. What is the ratio of the side lengths of the white rectangle?

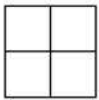
袋鼠國的國旗是一個長方形，被分成了三個同樣大小的長方形。問白色長方形的兩條邊的比例是多少？



- (A) 1 : 2      (B) 2 : 3      (C) 2 : 5      (D) 3 : 7      (E) 4 : 9

2. The numbers 1, 2, 3 and 4 are each written in different cells of the  $2 \times 2$  table. After that, the sum of the numbers in each row and column is calculated. Two of these sums are 4 and 5. What are the other two sums?

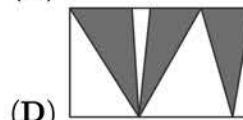
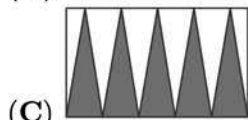
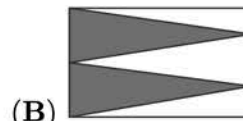
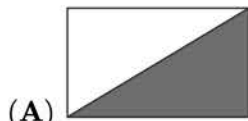
將數字1, 2, 3, 4填入 $2 \times 2$ 方格表的4個不同方格中，併計算各行與各列的總和。其中，兩個總和是4和5，問另兩個總和是多少？



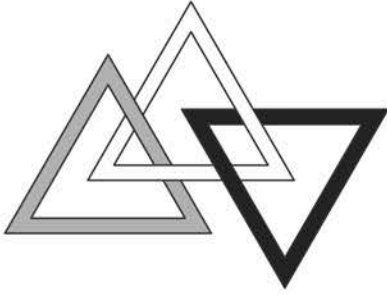
- (A) 6, 6      (B) 3, 5      (C) 4, 5      (D) 4, 6      (E) 5, 6

3. A rectangle has been shaded in five different ways as shown. In which diagram does the shaded part have the largest area?

一個長方形以如下五種不同的方式染色。問哪一個圖形中的陰影部分的面積最大？

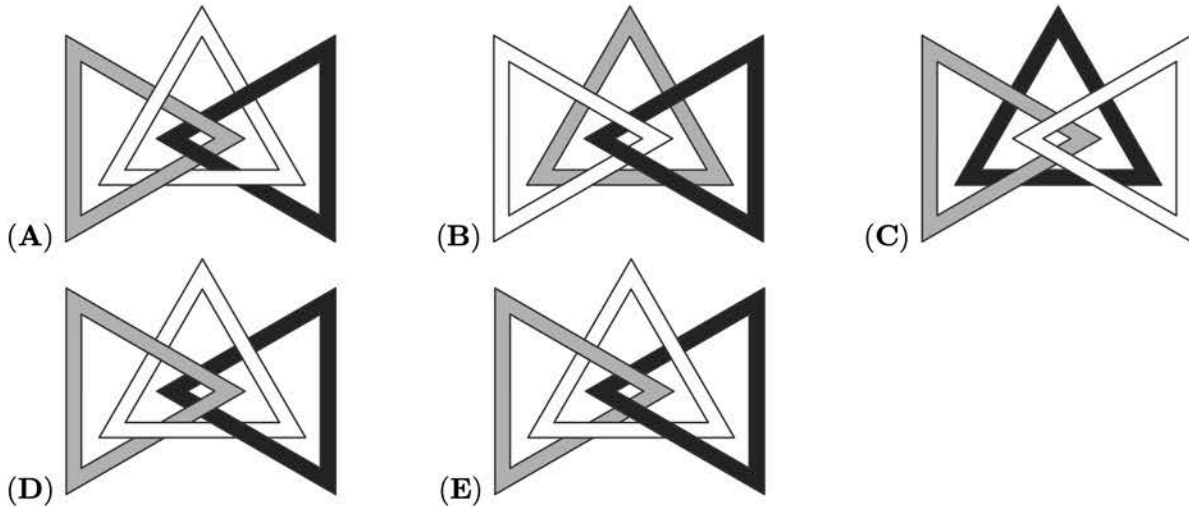


4.



Three triangles are linked as shown above. Which of the following pictures shows these three triangles linked in the same way?

如上所示，三個三角形相互纏繞。問下面哪個圖中的三個三角形有相同的纏繞方式？



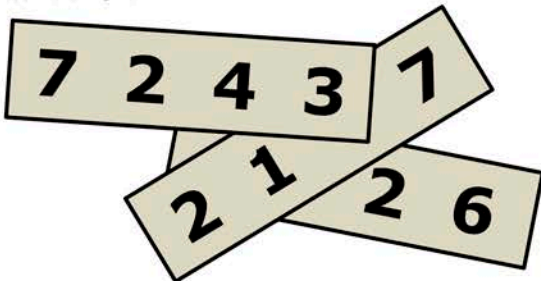
5. A pyramid has 23 triangular faces. How many edges does this pyramid have?

如果一個棱錐有23個三角形的面，問這個棱錐有多少條邊？

- (A) 23                      (B) 24                      (C) 46                      (D) 48                      (E) 69

6. Three 4-digit numbers are written on three pieces of paper as shown. The sum of the three numbers is 11126. Three of the digits are covered. What are the covered digits?

三張紙片上分別寫有三個四位數，並且這三數的和是11126。有三個數字被遮蓋住了。問被遮蓋住的數字是多少？



- (A) 1, 4, 7                      (B) 1, 5, 7                      (C) 3, 3, 3                      (D) 4, 5, 6                      (E) 4, 5, 7

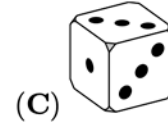
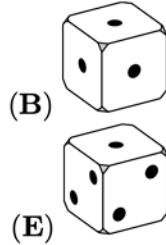
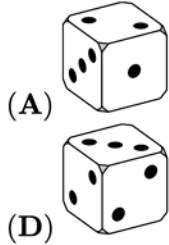
7. What is the first (leftmost) digit of the smallest positive integer whose digits add up to 2019?

最小的各位數字之和等於2019的正整數的首位（最左邊）數字是幾？

- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6

8. Each of the faces of a die is marked with either 1, 2 or 3 dots so that the probability of rolling a 1 is  $\frac{1}{2}$ , the probability of rolling a 2 is  $\frac{1}{3}$  and the probability of rolling a 3 is  $\frac{1}{6}$ . Which of the following cannot be a view of this die?

一個骰子的每個面上標有1個，2個，或者3個點，並且它擲出1的概率是 $\frac{1}{2}$ ，擲出2的概率是 $\frac{1}{3}$ ，擲出3的概率是 $\frac{1}{6}$ 。問下面哪個不可能是這個骰子的視圖？



9. Michael invented a new  $\diamond$  operation for real numbers, defined as  $x \diamond y = y - x$ . If  $a$ ,  $b$ , and  $c$  satisfy  $(a \diamond b) \diamond c = a \diamond (b \diamond c)$ , which of the following statements is necessarily true?

Michael定義了一個關於實數的新運算 $\diamond$ ，為  $x \diamond y = y - x$ 。如果 $a$ ， $b$ ，和 $c$ 滿足 $(a \diamond b) \diamond c = a \diamond (b \diamond c)$ ，問下面哪個論斷必然是正確的？

- (A)  $a = b$       (B)  $b = c$       (C)  $a = c$       (D)  $a = 0$       (E)  $c = 0$

10. How many of the numbers from  $2^{10}$  to  $2^{13}$ , inclusive, are divisible by  $2^{10}$ ?

在 $2^{10}$ 到  $2^{13}$ 之間（包括首尾二數），共有多少個數能被 $2^{10}$ 整除？

- (A) 2      (B) 4      (C) 6      (D) 8      (E) 16

**Part 2: 10 problems, 4 points each | 第二部分：10道題目，每題4分**

11. Which is the highest power of 3 dividing the number  $7! + 8! + 9!$ ?

能整除數 $7! + 8! + 9!$ 的3的最高方冪是多少？

- (A)  $3^2$       (B)  $3^4$       (C)  $3^5$       (D)  $3^6$   
 (E) a power of 3 higher than  $3^6$  一個比 $3^6$ 還高的3的方冪

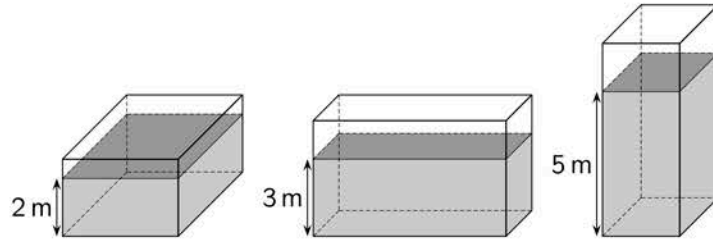
12. This year, the number of boys in my class has increased by 20% and the number of girls has decreased by 20%. We now have one student more than before. Which of the following could be the number of students in my class now?

今年，我的班級中男生的人數增加了20%，而女生的人數減少了20%。現在比以前多一個學生。問下列哪個數可能是現在班級裡的學生數？

- (A) 22      (B) 26      (C) 29      (D) 31      (E) 34



13.



A container in the shape of a rectangular box is partially filled with 120 cubic meters of water. The depth of the water is either 2 meters or 3 meters or 5 meters, depending on which side of the box is on the ground, as shown above (not to scale). What is the volume of the container in cubic meters?

一個長方體容器中裝了120立方米的水，但沒有裝滿。如上圖所示（未按比例繪製），取決於哪個面放在地上，不同擺放方式下的水深分別為2米、3米或5米。問此容器的體積是多少立方米？

- (A) 160                      (B) 180                      (C) 200                      (D) 220                      (E) 240

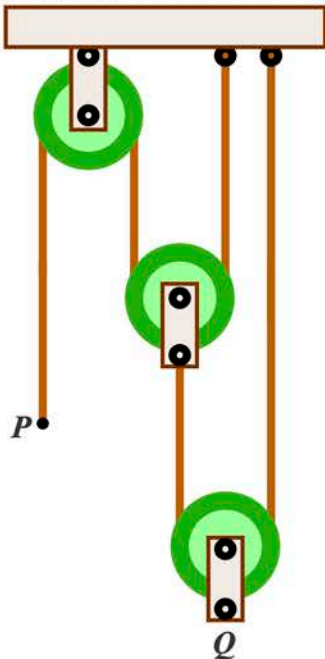
14. Three kangaroos, Alex, Bob and Carl, go for a walk every day. If Alex doesn't wear a hat, then Bob wears a hat. If Bob doesn't wear a hat, then Carl wears a hat. Today Carl is not wearing a hat. Who is certainly wearing a hat today?

三隻袋鼠Alex, Bob和Carl每天一起散步。如果Alex不戴帽子，那麼Bob會戴帽子。如果Bob不戴帽子，那麼Carl會戴帽子。今天Carl沒戴帽子。問誰今天一定戴帽子了？

- (A) only Alex and Bob 只有Alex和Bob                      (B) only Alex 只有Alex  
 (C) Alex, Bob and Carl Alex, Bob和Carl                      (D) neither Alex nor Bob 既不是Alex, 也不是Bob  
 (E) only Bob 只有Bob

15. The system shown consists of three pulleys with vertical sections of rope between them. The end  $P$  is moved down 24 centimeters. How many centimeters does point  $Q$  move up?

下面的滑輪系統是由三個滑輪和他們之間豎直的繩子組成。如果 $P$ 端被拉下24厘米，問 $Q$ 點會升高多少厘米？



- (A) 24                      (B) 12                      (C) 8                      (D) 6                      (E)  $\frac{24}{5}$

16. A positive integer  $n$  is called *good* if its largest divisor (excluding  $n$ ) is equal to  $n - 6$ . How many *good* positive integers are there?

一個正整數  $n$  被稱為“好數”如果它的最大的因數（不包括  $n$  本身）等於  $n - 6$ 。問一共有多少個正整數是“好數”？

- (A) 1 (B) 2 (C) 3  
(D) 6 (E) infinitely many 多

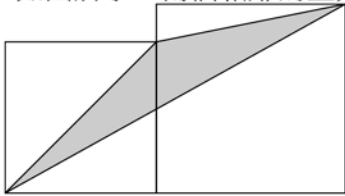
17. A box contains 4 chocolates and 1 fruit chew. John and Mary take turns drawing a treat out of the box without replacement. Whoever draws the fruit chew wins. John draws first. What is the probability that Mary wins?

一個盒子裡放著4包朱古力糖和1包水果糖。John和Mary輪流從盒子裡抽出一包糖，抽出的糖不再放回，誰先抽到水果糖誰就算贏。如果John先抽的話，Mary獲勝的概率是多少？

- (A)  $\frac{2}{5}$  (B)  $\frac{3}{5}$  (C)  $\frac{1}{2}$  (D)  $\frac{5}{6}$  (E)  $\frac{1}{3}$

18. Two adjacent squares with side lengths  $a$  and  $b$  ( $a < b$ ) are shown. What is the area of the shaded triangle?

如圖所示，兩個相鄰的正方形邊長分別是  $a$  和  $b$  ( $a < b$ )。問陰影三角形的面積是多少？



- (A)  $\sqrt{ab}$  (B)  $\frac{1}{2}a^2$  (C)  $\frac{1}{2}b^2$  (D)  $\frac{1}{4}(a^2 + b^2)$  (E)  $\frac{1}{2}(a^2 + b^2)$

19. What is the integer part of  $\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20}}}}}$ ?

$\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20}}}}}$  的整數部分是多少？

- (A) 4 (B) 5 (C) 6 (D) 20 (E) 25

20. To calculate the result of  $\frac{a+b}{c}$ , Sara types  $a + b \div c =$  on a calculator and the result is 11 ( $a$ ,  $b$ , and  $c$  are positive integers). She then types  $b + a \div c =$  and she is surprised to see that the result is 14. She realizes that the calculator is designed to calculate divisions before additions. What is the correct result of  $\frac{a+b}{c}$ ?

為了計算  $\frac{a+b}{c}$  的結果，Sara在計算器上輸入  $a + b \div c =$ ，並得到結果是11（ $a$ ， $b$ ，和  $c$  是正整數）。她接著又輸入了  $b + a \div c =$ ，並驚訝的發現結果是14。她隨即想起計算器是按照“先乘除，後加減”的順序計算的。問  $\frac{a+b}{c}$  的正確結果應該是多少？

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**Part 3: 10 problems, 5 points each | 第三部分：10道題目，每題5分**

**21.** Let  $a$  be the sum of all positive divisors of 1024 and  $b$  the product of all positive divisors of 1024. Then

設  $a$  是 1024 的所有正因數的總和， $b$  是 1024 的所有正因數的乘積。那麼有

- (A)  $(a - 1)^5 = b$     (B)  $(a + 1)^5 = b$     (C)  $a^5 = b$     (D)  $a^5 - 1 = b$     (E)  $a^5 + 1 = b$

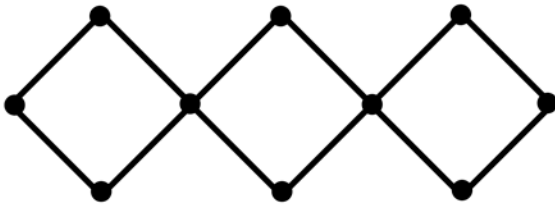
**22.** What is the set of all values of the parameter  $a$  for which the number of solutions of the equation  $2 - |x| = ax$  is equal to two?

問使得方程  $2 - |x| = ax$  的解的個數恰為兩個的所有參數  $a$  的集合是什麼？

- (A)  $(-\infty, -1]$     (B)  $(-1, 1)$     (C)  $[1, +\infty)$     (D)  $\{0\}$     (E)  $\{-1, 1\}$

**23.** The vertices of the network shown are labelled with the numbers from 1 to 10. The sum  $S$  of the four labels on each square is the same. What is the least possible value of  $S$ ?

下圖中網絡的各頂點處分別標有從 1 到 10 的數。假設每個正方形的四個頂點處所標數的總和都等於  $S$ 。問  $S$  的最小可能值是多少？



- (A) 18    (B) 19    (C) 20    (D) 21    (E) 22

**24.** How many planes pass through at least three vertices of a given cube?

問共有多少個平麵包含一個給定立方體的至少三個頂點？

- (A) 6    (B) 8    (C) 12    (D) 16    (E) 20

**25.** Four distinct straight lines pass through the origin of the coordinate system. They intersect the parabola  $y = x^2 - 2$  at eight points. What can be the product of the  $x$ -coordinates of these eight points?

有四條過原點的不同直線。它們和拋物線  $y = x^2 - 2$  交於 8 個點。問這 8 個點的  $x$  坐標的乘積可能是多少？

- (A) only 16 只能是 16    (B) only -16 只能是 -16  
 (C) only 8 只能是 8    (D) only -8 只能是 -8  
 (E) There are several possible products. 有很多種可能的乘積

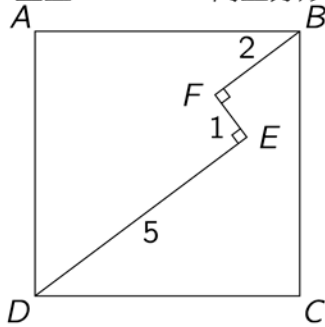
**26.** For how many integers  $n$  is  $|n^2 - 2n - 3|$  a prime number?

有多少個整數  $n$ ，使得  $|n^2 - 2n - 3|$  是素數？

- (A) 1    (B) 2    (C) 3    (D) 4  
 (E) infinitely many 無窮多

27. A path  $DEFB$  with  $DE \perp EF$  and  $EF \perp FB$  lies inside the square  $ABCD$  as shown. Given that  $DE = 5$ ,  $EF = 1$  and  $FB = 2$ , what is the length of the side of the square?

如圖所示，正方形 $ABCD$ 中有一條折線 $DEFB$ 滿足 $DE \perp EF$ ,  $EF \perp FB$ 。已知 $DE = 5$ ,  $EF = 1$ ，並且  $FB = 2$ ，問正方形的邊長是多少？



- (A)  $3\sqrt{2}$                       (B)  $\frac{7\sqrt{2}}{2}$                       (C)  $\frac{11}{2}$                       (D)  $5\sqrt{2}$   
 (E) none of the previous 不是以上的數

28. The sequence  $a_1, a_2, a_3, \dots$  starts with  $a_1 = 49$ . For  $n \geq 1$ , the number  $a_{n+1}$  is obtained by adding 1 to the sum of the digits of  $a_n$  and then squaring the result. Thus  $a_2 = (4 + 9 + 1)^2 = 196$ . Determine  $a_{2019}$ .

數列 $a_1, a_2, a_3, \dots$ 起始於 $a_1 = 49$ 。當 $n \geq 1$ 時， $a_{n+1}$ 等於 $a_n$ 的各位數字之和加1，然後再平方得到的結果。例如， $a_2 = (4 + 9 + 1)^2 = 196$ 。求 $a_{2019}$ 。

- (A) 121                      (B) 25                      (C) 64                      (D) 400                      (E) 49

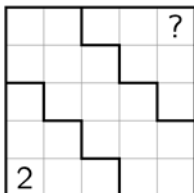
29. Three different numbers are chosen at random from the set  $\{1, 2, 3, \dots, 10\}$ . What is the probability that one of them is the average of the other two?

從集合 $\{1, 2, 3, \dots, 10\}$ 中隨機選取三個不同的數。其中的一個數恰好等於另外兩數的平均數的概率是多少？

- (A)  $\frac{1}{10}$                       (B)  $\frac{1}{6}$                       (C)  $\frac{1}{4}$                       (D)  $\frac{1}{3}$                       (E)  $\frac{1}{2}$

30. The square shown is filled with numbers in such a way that each row and each column contains the numbers 1, 2, 3, 4 and 5 exactly once. Moreover, the sum of the numbers in each of the three bold-bordered areas is equal. What number is in the upper right corner?

下圖的方格表的每行和每列中，數1, 2, 3, 4, 5恰好各出現一次。並且圖中的三塊以粗線為邊界的區域中的所有數的總和都相等。問右上角的那個數是幾？



- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5