NORD ANGLIA INTERNATIONAL SCHOOL DUBAI

A NORD ANGLIA EDUCATION SCHOOL

Primary Maths Calculation Policy 2015

Policy Statement

Aims

In light of the Department for Education Mathematics Programmes of study for Key Stage 1-2, the Calculation Policy has been created in order to reflect new concepts and pedagogy surrounding the teaching of mathematics. It is also designed to give pupils a consistent and smooth progression of learning in calculations throughout the school.

This policy has also been designed to include calculation stages which incorporate the requirements of children sitting an 11+.

(Please note that early learning in number and calculation in Reception/ FS 2 follows the 'Development Matters' EYFS document, and this policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.)

Raising Awareness of this Policy

We will raise awareness of this policy via:

- the Staff Handbook
- Staff Induction
- meetings with school personnel
- Headteacher reports to NAS Governance and The Parent Board

Monitoring the Effectiveness of the Policy

The practical application of this policy will be reviewed annually or when the need arises by the policy owner, the Principle and in line with NAS Corporate Governance.

Age Stage Expectations

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the stage that they are currently working at**, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Teachers are not limited to the age group that they are teaching and will frequently need to refer to lower or more advanced stages in order to support the needs of their students.

Providing a Context for Calculation

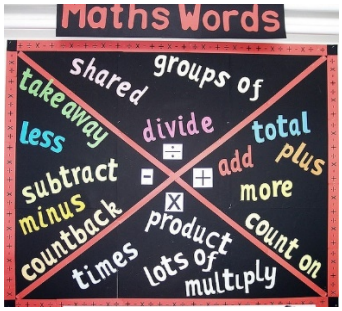
It is important that any type of calculation is given real life context or problems solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This should be a priority within calculation lessons.

Teaching of Mathematics

Children are introduced to the processes of calculation through **practical, oral** and **mental** activities. As teachers model alternative strategies, children will begin to understand the underlying ideas and develop ways of recording to support their thinking and calculation methods. They will use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved.

Over time children learn how to **use models and images,** such as empty number lines, to **support their mental and informal written methods of calculation**. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally.

Children will be continually reminded of the wide variety of mathematical terms that relate to each topic. **Maths vocabulary** should be clearly displayed in every classroom and referred to frequently.



Choosing a Calculation method

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:

They will do this by asking themselves:
Can I do this in my head?
Can I do this in my head using drawings or jottings?
Do I need to use a pencil and paper procedure?

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

[Mathematics Programmes of Study for Key Stages 1-2 2014]

The following should be considered when applying this policy

Children should not be made to go onto the next stage if:

- They are not ready.
- They are not confident.

The number line is a tool to support mental calculation. It is not a method and its use will depend on the calculation.

Every day is mental Maths day!
A mental method will always be the first port of call when tackling anything in Maths.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Stages in Addition

Stage 1: Add with numbers up to 20

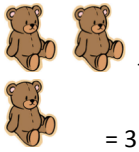
Use numbertracks, numberlines and practical resources to support calculation. Teachers demonstrate the use of the numberline. Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

Bead strings or bead bars can be used to illustrate addition by counting on 2 then counting on 1.

Children will explore *adding sets*

Harry brings 2 teddies to school. Lottie brings 1. How many teddies are at school today?

becomes



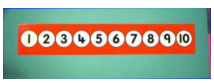
+

= 3

and *counting on* a set to another set.

There are three teddies. Phoebe comes to play and brings 2 more teddies. How many teddies do the children have to play with now?

+



3 + 2 = 5

Children should read, write and understand the addition (+) and equals (=) sign within number sentences.

Interpret addition number sentences and solve missing number problems, using concrete objects and number line addition to solve them:

7 + 4 = ☐ 6 + ☐ = 10 5 + 2 + 1 = ☐ ☐ + 2 = 6 ☐ + ☐ = 6

This builds on prior learning of adding by combining two sets of objects into one group in the early years.

KEY VOCAB:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key SKILLS for addition at Stage 1:

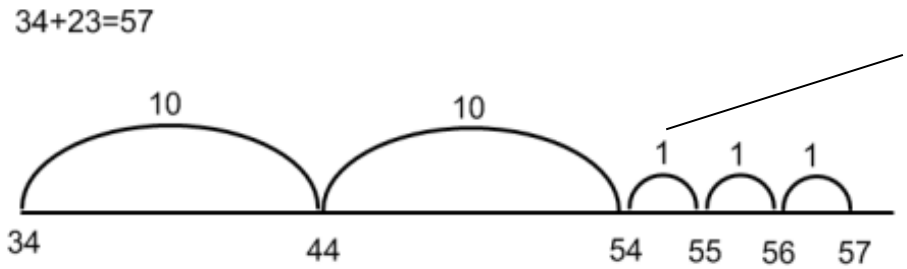
- Read and write numbers to 100 in numerals, incl. 1-20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve missing number problems
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Stage 2: Add with 2-digit numbers

Partitioning and Counting On:
Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

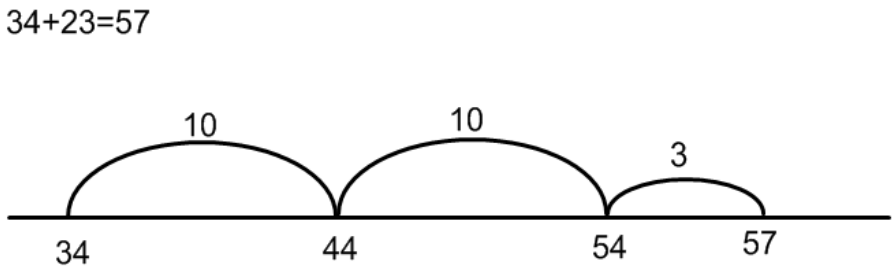
Initially children should practise partitioning into tens and ones before moving onto number line work. Eg: 23 = 10 + 10 + 10 + 1 + 1 + 1

First counting on in tens and ones.



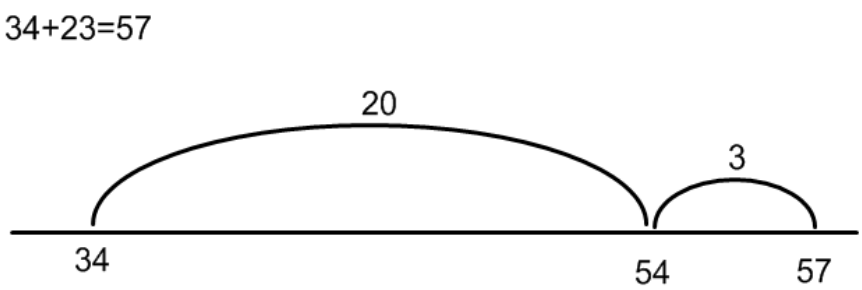
Initially jumps should *not* be labelled + or – to emphasise that number lines go both ways.

Then helping children to become more efficient by adding the ones in one jump.



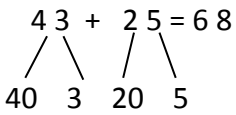
Followed by adding the tens in one jump and the ones in one jump.

Counting on from the largest number irrespective of the order of the calculation.
23+32= becomes 34+23=



Partitioning:

Partition numbers into tens and ones and recombine to give the answer.



40 + 20 = 60
3 + 5 = 8
60 + 8 = 68

Once secure, move on to calculations that bridge the tens:
4 8 + 2 3 = 40 + 8 + 20 + 3
= 40 + 20 = 60
= 8 + 3 = 11

60 + 11 = 71

Partitioned Column Method
STEP 1: (no crossing tens)

Move to this method once children are secure adding pairs of 2 digit numbers (as above.) This provides a simple introduction to column addition and utilises the skills acquired during stage 2.

34+23= 30 + 4
+ 20 + 3
50 + 7 = 57

Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: (crossing tens)

Once secure adding multiples of 10 to a 2-digit number mentally (80+11), children are ready for adding pairs of numbers that DO cross the tens boundary.

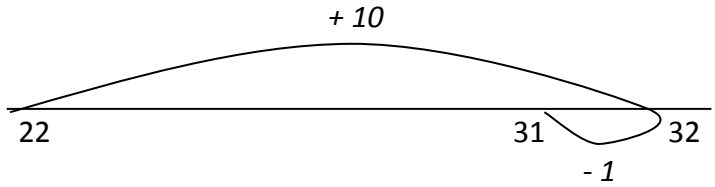
48+23= 40 + 8
+ 20 + 3
60 + 11 = 71

STEP 3: Children who are confident with this stage should move onto adding with 2 and 3 digit numbers using this method. They can also be introduced to the expanded addition method (see stage 3.)

Adding 9 or 11 by adding a 10 and adjusting by 1:

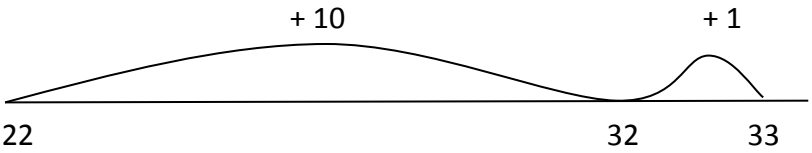
Add 9 by adding 10 and adjusting by 1 (less):

22 + 9 = 31



Add 11 by adding 10 and adjusting by 1 (more):

22 + 11 = 33



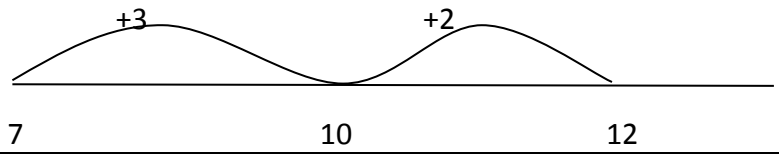
This can be extended to add 19 and 21 and adjusting.

Partitioning and Bridging through 10:

Steps in addition often bridge through a multiple of 10. Children can be taught this method in addition to counting on as it may develop into their preferred mental addition strategy.

Eg: Children should be able to partition the 5 to relate adding the 3 to make 10, then adding the 2.

7 + 5 = 12.



KEY VOCAB:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary

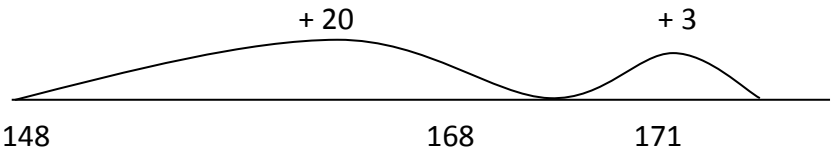
Key SKILLS for addition at Stage 2:

- Add a 2-digit number and ones (28+5)
- Add a 2-digit number and tens (28+30)
- Add pairs of 2-digit numbers (26+38)
- Add three single digit numbers (4+8+6)
- Show that adding can be done in any order (commutative law)
- Recall bonds to 20 and bonds of tens to 100 (40 + 60)
- Count in steps of 2, 3 and 5 and count in tens from any number
- Understand the place value of 2 digit numbers (tens and ones)
- Compare and order numbers to 100 using <> and = signs
- Read and write numbers to at least 100 in numerals and words
- Solve problems with addition (including missing numbers problems) using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Stage 3: Add numbers with up to 3 digits

Number lines may still be used by teachers to provide visual reinforcement of calculations.

Eg: 148 + 23 = 171



Develop Partitioned Column Method
(crossing tens) with up to 3 digits:

148+23=

100 + 40 + 8
\+ 20 + 3
100 + 60 + 11 = 171

Introduce the EXPANDED COLUMN ADDITION method:
The expanded addition method prepares children for the compact method and should be practiced regularly during this stage. Start with adding 2 digits then 3 digit numbers.

47
\+ 76
13
110
123

Add the ones first, in preparation for the compact method

Children who are very secure with 3 digit expanded column addition should be moved into the **compact column addition** method (see stage 4) and be introduced to 'carrying'.

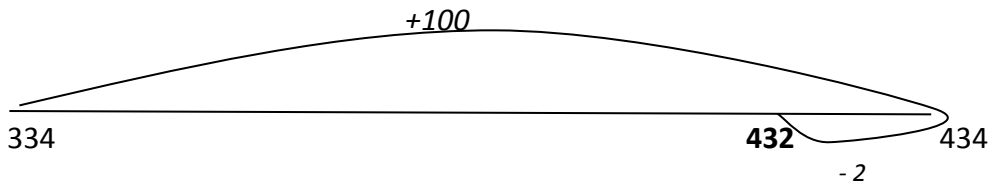
Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

KEY VOCAB:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact

Key SKILLS for addition at Stage 3:
- Add 2-digit numbers mentally, incl. those exceeding 100
- Add a 3-digit number and ones mentally (128+5)
- Add a 3-digit number and tens mentally (128+30)
- Add a 3-digit number and hundreds mentally (128+300)
- Read and write numbers to 1000 in numerals and words
- Estimate answers to calculations, using inverse to check answers
- Solve problems, including missing numbers problems, using number facts, place value and more complex addition
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones)
- Continue to practise a wide range of **mental addition strategies**, ie. Number bonds, using near doubles, partitioning and recombining, adding the nearest multiple of 10, 100 and adjusting.

Teachers may need to secure mental methods by using a *number line* to model these, for example:
Adding Near Multiples of 10 or 100.

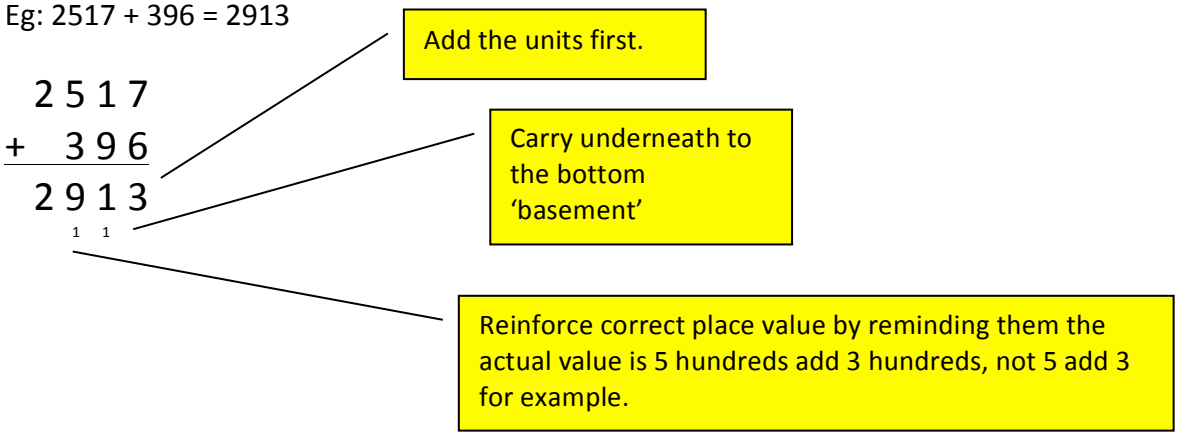
Eg: 334 + 98 = 432



Stage 4 – Add numbers with up to 4 digits

Once children are secure in adding up to 3 digits using the expanded addition method, move on to the **compact column method**, adding ones first, and carrying numbers underneath the calculation. Also include money and measures contexts.

Eg: 2517 + 396 = 2913



Introduce the compact addition method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition – see stage 3.) Teacher models the compact method with carrying, asking children to discuss similarities and establish how it is carried out.)

KEY VOCAB:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse.

Key SKILLS for addition at Stage 4:
- Select most appropriate method: mental, jottings or written and explain why
- Recognise the place value of each digit in a four-digit number
- Round any number to the nearest 10, 100 or 1000
- Estimate and use inverse operations to check answers
- Solve 2-step word problems in context, deciding which operations and methods to use and why
- Find 1000 more or less than a given number
- Continue to practise a wide range of ***mental addition strategies*** ie: add to the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining
- Add numbers with up to 4 digits using the formal written compact method of column addition

Stage 5 Add numbers with more than 4 digits

Including money, measures and decimals with different numbers of decimal places.

```
  £23. 59
+ £7. 55
 £31. 14
   1 1  1
```

The decimal point should be aligned in the same way as other place value columns, and must be in the same column in the answer.

```
  32359
+  7055
  49414
    1 1
```

Numbers should exceed 4 digits.

```
  19. 01
   3. 65
+  0. 70
  23. 36
   1 1
```

Pupils should be able to add more than two values, carefully aligning place value columns.

Children should understand the place value of **tenths** and **hundredths** and use this to align numbers with different numbers of decimal places.

KEY VOCAB:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths,

Key SKILLS for addition at Stage 5:
- Add numbers mentally with increasingly large numbers, using and practising a range of ***mental strategies*** ie: add the nearest multiple of 10, 100 and 1000 and adjust; use near doubles, inverse, partitioning and re-combining
- Use rounding to check answers and accuracy
- Solve multi-step problems in contexts, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- Add numbers with more than 4 digits using formal written method of column addition

Stage 6 Add several numbers of increasing complexity

```
  23. 361
   9. 080
  59. 770
+  1. 300
  93. 511
   2 1  2
```

Empty decimal places can be filled with zeros to show the place value in each column.

Adding several number with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.
- Adding several numbers with more than 4 digits.

KEY VOCAB:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones/ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths,

Key SKILLS for addition at Stage 6:
- Perform ***mental calculations***, including with mixed operations and large numbers, using and practising a range of mental strategies
- Solve multi-step problems in context, deciding which operations and methods to use and why
- Use estimation to check answers to calculations and determine levels of accuracy, in the context of a problem
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity

Stages in Subtraction

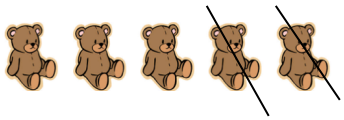
Stage 1 Subtract from numbers up to 20

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc. They use numberlines, numbertracks and practical resources to support calculation.

Children will explore the two models of subtraction –

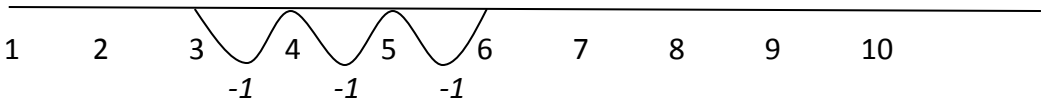
1. **Taking away/ counting back**
   (cross out to model subtraction)

Five teddy bears are at the park. 2 of them go home for their tea. How many bears are left at the park?



5 – 2 = 3

Count back in ones on a numbered number line to take away, with numbers up to 20.
6 – 3 = 3



2. **Finding the difference:**

Nine children were playing on the swings and three were playing on the slide. How many more were playing on the swings?

9 – 3 = 6

This will be introduced practically with the language - find the difference between, how many more – in a range of familiar contexts.
Eg: I am 6 years older than my brother. 9 is 6 more than 3.

Missing Number Problems:

Solve missing number problems, using concrete objects and number line addition:

7 - 4 = ☐ 6 - ☐ = 2 ☐ - 2 = 6 ☐ = 10 – 6 5 = ☐ - 2 5 = 7 - ☐

Children should read, write and understand the subtraction (-) and equals (=) sign within number sentences.

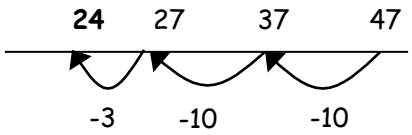
KEY VOCAB:
Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is..

Key SKILLS for subtraction at Stage 1:
- Given a number, say one more or one less
- Count to and over 100, forward and back, from any number
- Represent and use subtraction facts to 20 and within 20
- Subtract with one digit and two digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie: bead string, objects, cubes) and pictures, and missing number problems
- Read and write number from 0 to 20 in numeral and words

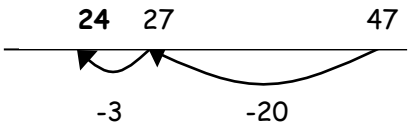
Stage 2 Subtract with 2- digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.
Use this strategy for –
2 digit number subtract ones (45 – 7)
2 digit numbers subtract tens (33 – 20)
Subtracting pairs of 2 digit numbers (as below)

First counting back in tens and ones:
47 - 23 = 24



Then move back in more efficient jumps on a number line.
47 - 23 = 24

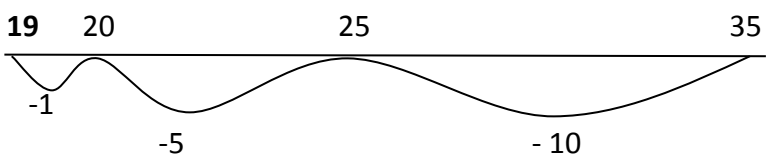


Combine methods with use of a hundred square to reinforce understanding of number value and order.

Partitioning:
The subtraction calculation above requires children to subtract a single digit number or a multiple of 10 from a two digit number using a number line. Once children are secure they can be encouraged to complete calculations without a number line, using their mental subtraction skills.

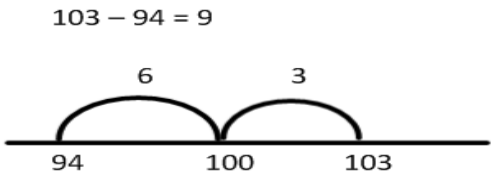
Eg: 74 – 27 =
74 – 20 = 54
54 - 7 = 47

Bridging through 10:
Steps in subtraction often bridge through a multiple of 10. Children can be taught this method in addition to counting back as it may develop into their preferred mental subtraction strategy. (Partition the second number only)
Eg: 35 - 16 =
35 – 10 - 5 - 1 = 19

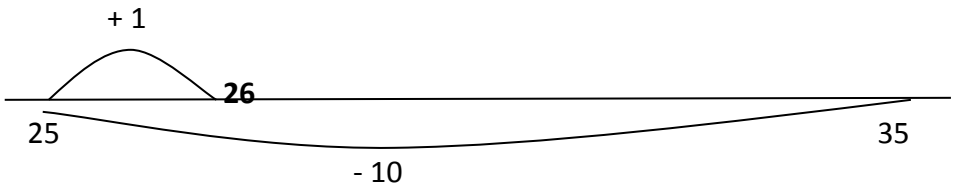


Counting on:
Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction.

Start with the **smaller** number and count on to the largest.



Subtract 9 or 11 by subtracting a 10 and adjusting by 1:
35 – 9 = 26



This can be extended to subtract 19 and 21.

KEY VOCAB:
Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is.. difference, count on, strategy, partition, tens, ones

Key SKILLS for subtraction at Stage 2:
- Recognise the place value of each digit in a two digit number
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally
- Show that subtraction of one number from another cannot be done in any order
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems
- Continue to solve missing number calculations with 1 and 2 digit numbers
Eg: 20 – D = 15 or 15 + 5 = 20 - D
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation and also applying their increasing knowledge of mental and written methods

Stage 3 Subtract with 2 and 3 digit numbers

Introduce **partitioned column** subtraction method.

Step 1: introduce this method where no 'taking' from the tens or hundreds is required

89 – 57 = 32

80 + 9
\- 50 + 7
30 + 2 = 32

Step 2: introduce 'taking' through practical subtraction using cubes or Base 10.

Make the larger number with the Base 10, then subtract the smaller from it.
For eg: 72 – 47. Make 72 (7 x rods of 10, 2 ones.)
Before subtracting 7 from the 72 blocks, they will need to 'take' a 10 from the 10s. Then they will be ready to subtract the 7 ones, then the 4 tens.

Then practise **partitioned column** subtraction method with 'taking':

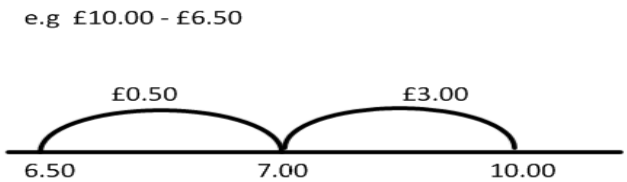
60
~~7~~0 + ¹2
\- 40 + 7
20 + 5 = 25

Step 2: When children are secure with the understanding of taking from the tens, they can use the partitioned column method to subtract any 2 and 3 digit number.

600 140
~~7~~~~0~~ + ~~5~~0 + ¹4
\- 80 + 6
600 + 60 + 8 = 668

Mental Strategy for Subtraction – Counting on

Continue to reinforce counting on as a strategy for close together numbers (eg: 121 – 118) and also for numbers that are nearly multiples of 10, 100, 1000 or £s, which make it easier to count on (eg: 102 – 89, 131 – 79, or calculating change from £1)



KEY VOCAB:
Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is.. difference, count on, strategy, partition, tens, ones, taking, decrease, hundreds, value, digit
Vocabulary note: - The terms exchanging, trading, decomposition, borrowing and taking refer to the same method of subtraction. NAS teachers have agreed to use the term 'taking**' as this can easily be reinforced during practical and role play work. Students will learn that they 'take' from their larger neighbour in order to be able to complete a calculation. As they do not give back anything there is no obvious exchanging or trading being carried out. **

Key SKILLS for subtraction at Stage 3:
- Subtract mentally a: 3-digit number and ones, 3 – digit number and tens, 3- digit number and hundreds
- Estimate answers and use inverse operations to check
- Solve problems, including missing number problems
- Find 10 or 100 more of less than a given number
- Recognise the place value of each digit in a 3- digit number
- Know when to count up differences as a mental strategy when numbers are close together or near multiples of 10
- Read and write numbers up to 1000 in numerals and words
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (eg: subtracting 19 to 21) and select most appropriate methods to subtract, explaining why

Stage 4 Subtract with up to 4 - digit numbers

Partitioned column subtraction with taking:

2754 – 1562 = 1192

600 *150*

2000 + ~~7~~00 + ~~5~~0 + 4

-1000 + 500 + 60 + 2

1000 + 100 + 90 + 2

Once secure move on to **compact column subtraction** -

5 13 1

~~6~~~~4~~67

- 2684

3783

To introduce the compact method, ask the children to perform a subtraction calculation with the familiar partitioned column subtraction. Then display and compare the compact version. Review with pupils, compare and contrast to develop understanding.

Give plenty of opportunities to apply these strategies to money and measures.

KEY VOCAB:
Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is.. difference, count on, strategy, partition, tens, ones, taking, decrease, hundreds, value, digit, inverse

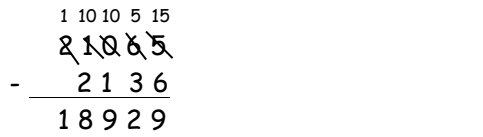
Key SKILLS for subtraction at Stage 4:
- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc
- Children select the most appropriate and efficient methods for given subtraction calculations
- Estimate and use inverse operations to check answers
- Solve addition and subtraction 2 – step problems, choosing which operations and methods to use and why
- Solve simple measure and money problems involving fractions and decimals to two decimal places
- Find 100 more or less than a given number
- Count backwards through zero, including negative numbers
- Recognise place value of each digit in a 4-digit number
- Round any number to the nearest 10, 100, 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers

Stage 5 Subtract with at least 4-digit numbers

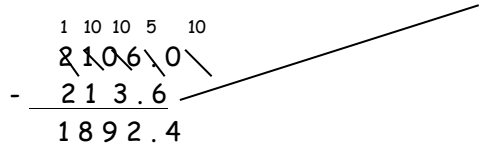
(including money, measures and decimals)

Compact column subtraction

(with taking)



Practise repeated taking like this and focus on when there is a 0 and they need to 'knock next door' to take. This makes a 10, not a 1. See example.
If children not secure with place value remain in the partitioned column method until ready.



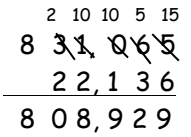
Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point. (Add a '0' in any empty decimal place to aid understanding of what to subtract in that column)

Create lots of opportunities for subtracting and finding differences with money and measures.

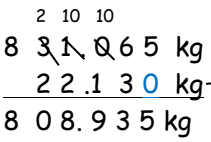
KEY VOCAB:
Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is.. difference, count on, strategy, partition, tens, ones, taking, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

Key SKILLS for subtraction at Stage 5:
-Subtract numbers mentally with increasingly large numbers
-Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy
-Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why
-Read, write, order and compare numbers to at least 1 million and determine the value of each digit
-Count forwards or backwards in steps of powers of 10 for any given number up to 1 million
-Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0
-Round any number up to 10 million to the nearest 10, 100, 1000, 10 000 and 100, 000.

Stage 6 Subtracting with increasingly large and more complex numbers and decimal values



Use the compact column method to subtract money and measures, including decimals with different numbers of decimal places.



Empty decimal places can be filled with 0 to show the place value in each column

KEY VOCAB:

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is.. difference, count on, strategy, partition, tens, ones, taking, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

Key SKILLS for subtraction at Stage 6:

-Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why
-Read, write, order and compare numbers up to 10 million and determine the value of each digit
-Round any whole number to a required degree of accuracy
-Use negative numbers in context, and calculate intervals across zero
-Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

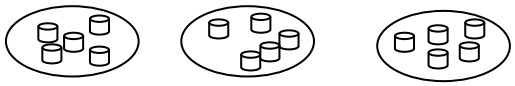
Stages in Multiplication

Stage 1 – Multiply with concrete objects, arrays and pictorial representations

- Give children experience of counting equal groups of objects in 2s, 5s and 10s
- Present practical problem solving activities involving counting equal sets or groups.

Eg: How many arms will 4 teddies have? 

Eg: There are 5 sweets in one bag. How many sweets are in 3 bags altogether?



5 + 5 + 5 = 15

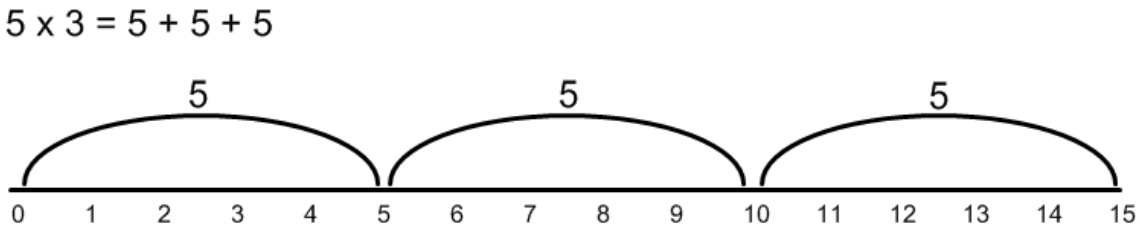
KEY VOCAB:
Groups of, lots of, times, array, altogether, multiply, count

Key SKILLS for multiplication at Stage 1:
-Count in multiples of 2, 5 and 10
-Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
-Make connections between arrays, number patterns and counting in twos, fives and tens
-Begin to understand doubling using concrete objects and pictorials representations

Stage 2

Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Use Repeated addition on a number line:



Starting from 0, make equal jumps on a number line to work out multiplication facts and write multiplication statements using X and = signs. 5 X 3 = 15

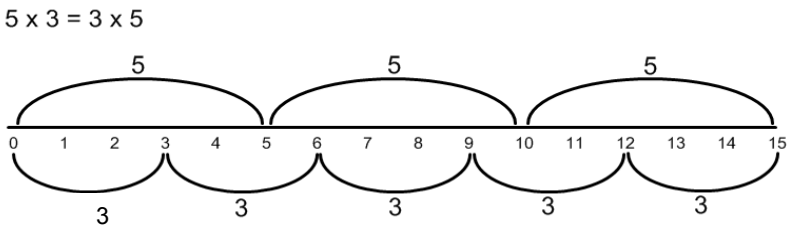
Use arrays:

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method and an understanding of the communtative law.

5 x 3 = 15 3 x 5 = 15

Commutativity:

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line as well as in arrays (as above) if children need further reinforcement.



Missing Number Problems:

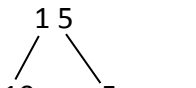
Solve missing number problems, using concrete objects and number line addition to solve them:

7 x 2 = ☐ 6 x ☐ = 12 ☐ x 2 = 16 ☐ = 10 x 5 10 = ☐ x 2 15 = 3 x ☐

Children should read, write and understand the multiplication (x) and equals (=) sign within number sentences.

Partitioning and Doubling:
Children need to be secure with partitioning numbers into 10s and 1s to allow them to double easily. Children should regularly practise doubling numbers and should work towards learning all doubles of numbers up to and including 15.

Eg: Double 15?



10 + 5
20 + 10 = 30

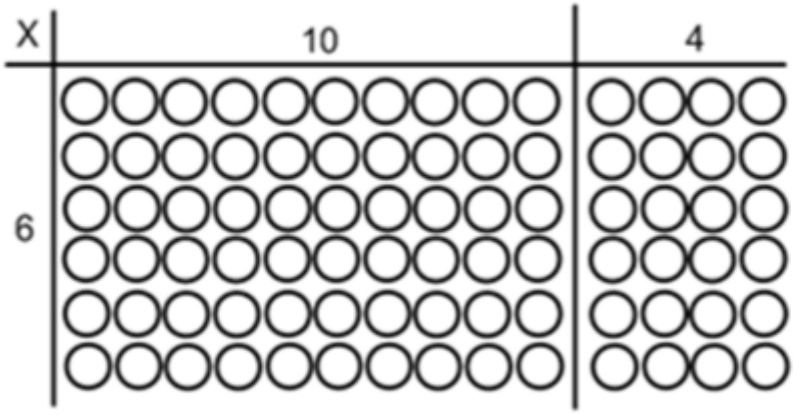
KEY VOCAB:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..

Key SKILLS for multiplication at Stage 2:
-Count in steps of 2, 3 and 5 from zero and in 10s from any number
-Recall and use multiplication facts from the 2, 5 and 10 multiplication tables and other tables facts if confident (Once division has been introduced children should also be able to derive division facts for learnt multiplication facts.)
-Write and calculate number statements using the X and = signs
-Show that multiplication can be done in any order (commutative law)
-Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods and multiplication facts

Stage 3. Multiply 2 Digits by a Single Digit

Introduce the **grid method** by physically making an array to represent the calculation (eg: make 6 lots of 14 with 10s and 1s in Base 10,) then translate this to an array grid –

14 x 6 = 84



Practise Partitioning:
(In order for children to use the grid method they need to be able to partition numbers into tens and ones and multiply multiples of 10 by a single digit. 'Remove the 0, multiply and add 0 back on. Eg: 30 x3 = 3x3 = 9, add 0 back on 90)

38 x 3 = 30 x 3 + 8 x 3

90 + 24 = 114

Encourage children to work out unknown multiplication facts by repeated addition or other taught mental strategies (eg. Commutative law, working out near multiples and adjusting, using doubling etc)

Grid method – TU x U

23 x 8

X	20	3
8	160	24

160 + 24=184

Encourage children to add up mentally first but then they may need a suitable written method if they cannot add mentally.

Those children that are ready should move on to use **short multiplication methods.** (Children should be confident in times tables, accurate at multiplying using the grid method and should be confident in 'carrying' for written column addition before doing this.)

Eg:

$$\begin{array}{r} 23 \\ \times \quad 8 \\ \hline 184 \\ \hline {\scriptstyle 2} \end{array}$$

KEY VOCAB:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..partition, grid method, multiple, product, tens, ones, value

Key SKILLS for multiplication at Stage 3:
- Recall and use multiplication facts for the 2, 3 ,4, 5, 8 and 10 times tables and multiply multiples of 10. Go on to 6, 9, 11 and 12 times tables if confident
- Continue to derive division facts from new multiplication tables
-Solve multiplication problems, including missing number problems
☐ x 5 = 20 3 x △ = 18 ☐ x O = 32
-Develop mental strategies using commutativity (eg. 4x12x5 = 4x5x12 = 20 x 12 = 120)
-Solve simple problems in contexts, deciding which operations and methods to use

Stage 4. Multiply 2 and 3 Digits by a Single Digit

(using multiplication tables up to 12 x 12)

Continue to develop the grid method:

136 x 5 = 680

X	100	30	6
5	500	150	30

500
150
+ 30
680

Encourage column addition to add accurately.

USE YOUR FACTS:
Encourage children to utilise their learnt multiplication facts to help them solve parts of the grid.
grid.
Eg: X 5 is the same as X 10 ÷ 2
X 50 is the same as X 100 ÷ 2
X 25 is the same as X 100 ÷ 2

Continue to develop short multiplication skills:

136
X 9
1224
3 5

Spend time comparing both the grid method and short multiplication method.

Children should:

- Approximate before they calculate and make this a regular part of their calculating going back to approximation to check the reasonableness of their answer.
Eg: 346x9 is approx. 350x10 = 3500 (record approximation to compare with final answer)
-Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

KEY VOCAB:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..partition, grid method, multiple, product, tens, ones, value, approximate

Key SKILLS for multiplication at Stage 4:
-Count in multiples of 6, 7, 9, 25, 50, 100 and 1000
-Recall multiplication facts for all multiplication tables up to 12 x 12
-Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100. They should be able to do this with 1-3 digit numbers.
-Use commutativity and other strategies mentally
3x6 = 6 x 3, 2 x 6 x 5 = 10 x 6, 39 x 7 = 30 x 7 + 9 x 7

Stage 5. Multiply up to 4 digits by 1 or 2 digits

Continue to practise the grid method and short/column multiplication skills for multiplying by a single digit.
Ask children to compare each and unpick the steps.
Develop this by multiplying 4 digits by a single digit.

Eg: 1362 x 5 = 6800

X	**1000**	**300**	**60**	**2**
5	500	150	30	10

```
  1 3 6 2
    X   9
-------
1 2 2 5 8
  3 5 1
```

Introduce long multiplication for multiplying by **2 digits**:

Step 1: **Expanded Long Multiplication**

```
      2 3 5
    X   7 4
-----------
        2 0
      1 2 0
      8 0 0
      3 5 0
    2 1 0 0
+   1 4 0 0
-----------
    4 7 9 0
```

Children who are more confident multiplying larger numbers, should be moved onto using the grid method for long multiplication.

Step 2: **Grid Method Long Multiplication**

X	**10**	**8**	
10	100	80	180
3	30	24	54

Teachers should compare the grid method with expanded, long multiplication methods:

```
  1 8
X 1 3
-----
  5 4
  2
1 8 0
-----
2 3 4
```

1. 3 x 8 = 24 (carry the 2)
   3 x 10 = 30
   Add 2 carried = 50. Ans: 54

2. 10 x 8 = 80
   10 x 10 = 100. Ans: 180

Move toward more complex numbers:

Using column/short multiplication if multiplying by a **single** digit:

```
  3 6 5 2
X       8
2 9 2 1 6
  5 4 1
```

Using expanded/ long multiplication if multiplying by **two** digits:

```
   1 2 3 4
X      1 6
   7 4 0 4 (1234 x 6)
     2 2
1 2 3 4 0  (1234 x 10)
1 9 7 4 4
```

KEY VOCAB:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..partition, grid method, multiple, product, tens, ones, value, approximate, factor, integer, decimal, short/long multiplication, carry,

Key SKILLS for multiplication at Stage 5:
-Identify multiples and factors using knowledge of multiplication tables to 12 x 12
-Solve problems where larger numbers are decomposed into their factors
-Multiply and divide integers and decimals by 10, 100 and 1000
-Recognise and use square and cube numbers and their notation
-Solve problems involving combinations of operations, choosing and using calculations and methods appropriately

Stage 6. Develop short and long multiplication as in Stage 5 and Multiply decimals with up to 2 decimal points by a single digit

Short multiplication Development:

Decimals -

```
  3 . 1 9
X     8
2 5 . 5 2
  1   7
```

- Line up decimal points in the question and the answer
- Remind children that the single digit (8) belongs in the units column
- This works well for multiplying money and other measures

Use rounding and place value to make approximations before calculating and use these to check answers against.
Eg: 3.19 x 8…. 3 x 9 = 27, answer should be close to 27.

More than 4 digits by a single digit –

```
    2 3  6  5 2
X              8
1 8 9  2  1 6
  2 5  4  1
```

Long multiplication Development: (multiply numbers with at least 4 digits by 2 digits)

```
    5 1 2  3 4
  X        1 6
  3 0 6  4 0 4  (51234 x 6)
        2  2
+ 5 1 2  3 4 0  (51234 x 10)
  8 1 9  7 4 4
```

KEY VOCAB:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..partition, grid method, multiple, product, tens, ones, value, approximate, factor, integer, decimal, short/long multiplication, carry, tenths, hundredths, decimal

Key SKILLS for multiplication at Stage 6:
- Recall multiplication facts for all times tables facts
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication
-Perform mental calculations with mixed operations and large numbers
-Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods
-Estimate answers using rounding and approximation
-Round any integer to a required degree of accuracy

Stages in Division

Stage 1 Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

Eg: How many groups of 4 can be made with 12 faces? = 3
Or, What is 12 shared between 3? = 4

Teach children to understand the difference between grouping objects (How many groups of 2 can you make?) and sharing (Share these sweets between 2 people.)

Children will need to count in multiples of 2s, 5s and 10s

Encourage children to find half of a group of objects by sharing into 2 equal groups.

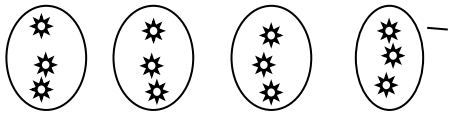
KEY VOCAB:
Share, share equally, one each, two each, group, groups of, lots of, array

Key SKILLS for division at Stage 1:
-Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representation arrays with the support of the teacher
-Through grouping and sharing small quantities, pupils begin to understand division and finding simple fractions of objects, numbers and quantities
-They make connections between arrays, number patterns and counting in twos, fives and tens

Stage 2 – Group and share, using the ÷ and + sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

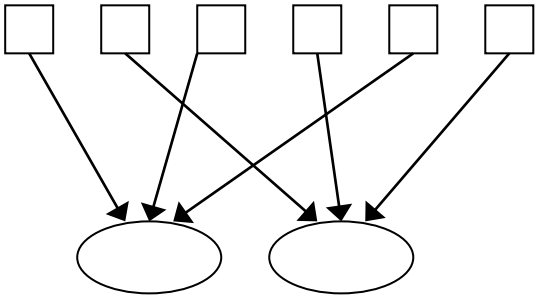


- This represents 12 ÷ 3 = 4
- How many groups of 3 are there? = 4
- Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally

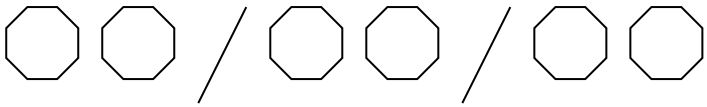
Sharing and Grouping:

Children should be taught to recognise whether problems require sharing or grouping.

Sharing: 6 sweets shared between 2 people, how many do they each get? = 3 sweets each



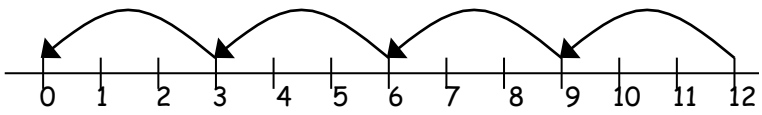
Grouping: There are 6 sweets. How many people can have 2 sweets each? 3 people



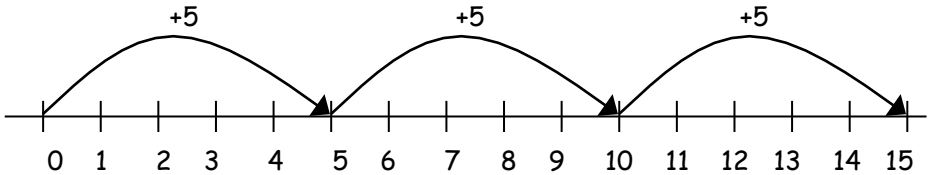
Ensure the children have lots of practical experiences completing these questions.

Repeated subtraction or addition using a number line or bead bar:

12 ÷ 3 = 4 (repeated subtraction)



15 ÷ 5 = 3 (repeated addition)



Provide lots of real life style questions. Eg: A CD costs £3, how many can I buy with 12?
I.e: How many groups of 3 are there in 12?

Continue to give children practical apparatus to work out these problems.

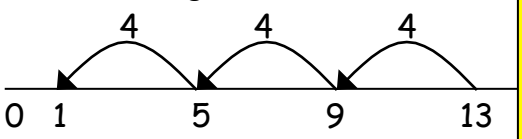
Children may find it easier to use repeated addition at stage 2 as they will be more confident adding than subtracting at this stage.

KEY VOCAB:
Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over,

Key SKILLS for division at Stage 2:
-Count in steps of 2, 3 and 5 from 0
-Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
-Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the X, ÷ and = signs
-Know that multiplication can be done in any order (commutative) and division of one number by another cannot
-Solve problems involving multiplication and division, using materials, arrays, repeated addition or subtraction, mental methods, and multiplication and division facts, including problems in context.

Stage 3 Divide 2 Digit numbers by a single digit

Step 1: Develop division skills by grouping on a number line. Can now teach the concept of **remainders**. Eg: 13 ÷ 4 = 3 r 1



This should be introduced practically with arrays as well as a number line. Children should work towards calculating basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method later.

Step 2: **Introduce short division** (no remainders)
Carefully select the numbers so that there is no carrying or remainders.

$$3\overline{)96}\quad 32$$

Remind children of correct place value, that 96 is equal to 90 and 6, but in short division ask:

- How many 3's in 9? = 3 record above
- How many 3' in 6? = 2, record above

Step 3: **Short division,** with carrying but no remainders in final answer
(This should only be taught once children can calculate remainders using number line)

$$3\overline{)7^{1}2}\quad 24$$

- How many 3's in 7? = 2, remainder/carry 1
- How many 3's in 12? = 4.
- Answer recorded on top = 24

If children exceed at this level, then move them on to Stage 4.

KEY VOCAB:
Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple

Key SKILLS for division at Stage 3:
-Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s)
-Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
-Solve problems, in contexts, and including missing number problems, involving multiplication and division
-Use efficient mental methods to solve problems. E.g: using multiplication and division facts (using 3 x 2 = 6, 6 ÷ 3 and 2 = 6 ÷ 3) to derive related facts (30 x 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷3)

Stage 4 Divide up to 3 Digit numbers by a single digit

Continue to Develop Short division, with carrying but no remainders in the final answer initially

```
  2 4
3 )7 12

  2 2 4
3 )6 7 12
```

Practise dividing 2 digit numbers then move onto dividing numbers with up to 3 digits. Calculations should not result in a final answer with remainder.

Practise place value, when answer for first column is zero

```
  0 9 1
3 )2 27 3
```

When the answer for the first column is zero, children should initially write a zero above to acknowledge its place but must always 'carry' the number (2) over to the next digit as a remainder

Short Division with Remainders

```
  0 9 0 r 2
3 )2 27 2
```

If children exceed at this expectation, move them on to Stage 5

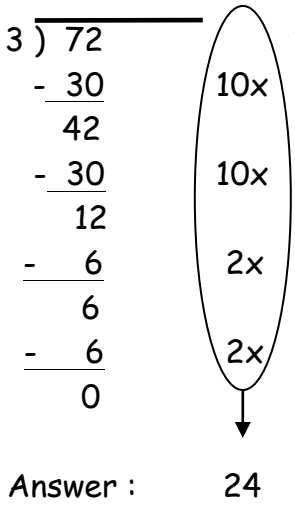
KEY VOCAB:
Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor,

Key SKILLS for division at Stage 4:
-Recall multiplication and division facts for all numbers up to 12 x 12
-Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1000
-Be fluent in the formal written method of short division with exact answers
-Use mental methods and extend this to three digit numbers to derive facts. Eg: 200 x 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers.

Stage 5 Divide up to 4 Digit numbers by a single digit

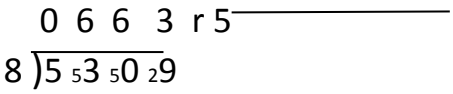
Introduce Chunking, when dividing by single digits.

72 ÷ 3



Although chunking is part of teaching long division it is **important** to introduce children to the concept at this stage.
If they are able to chunk when dividing by a single digit they will approach long division/ dividing numbers by 2- digits with more confidence.

Develop Short division skills: (including remainders)



Develop different ways of presenting answers:
The answer to 5309 ÷ 8 could be expressed as

- 663 and five eighths,
- 663 r 5
- as a decimal
- rounded as appropriate to the problem

Once pupils have examples that give rise to remainder answers, division needs to have a **real life problem solving context**, where pupils consider the meaning of the remainder and how to express it in their answers.

Ensure questions include money and measures in context.

KEY VOCAB:
Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite numbers

Key SKILLS for division at Stage 5:
-Recall multiplication and division facts for all numbers up to 12 x 12 (as in Stage 4)
-Multiply and divide numbers mentally, drawing on known facts
-Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
-Solve problems involving multiplication and division where larger numbers are decomposed into their factors
-Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
-Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
-Work our whether a number up to 100 is prime, and recall prime numbers to 19
-Use multiplication and division as the inverse
-Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding
Eg: 98 ÷ 4 = 24 r 2 = 24 and 2 quarters OR 24 ½ = 24.5 rounded = 25.0

Stage 6 Divide at least 4 digits by both single-digit and 2-digit numbers

(including decimal numbers and quantities)

Short Division, for dividing by a single digit: eg: 6497 ÷ 8

Calculating a Decimal Remainder:

$$8\overline{)6\,^{6}4\,9\,^{1}7.\,^{1}0\,^{2}0\,^{4}0} \quad 0\;8\;1\;2.1\;2\;5$$

Rather than expressing the remainders as r 1, a decimal point is added after the ones because there is still a remainder, and the one remainder is carried onto the zeros after the decimal point (to show there was no decimal value in the original number.)
Keep dividing to an appropriate degree of accuracy for the problem being solved.

$8\overline{)6\;4\;9\;7.\mathbf{0\;0\;0}}$

When introducing this method, present the question with the decimal point and zeros in place so they know what is expected. Once secure this will not be necessary.

Long Division by chunking, for dividing by 2 digits: eg: 972 ÷ 36

Find out how many 36s are in 972? By subtracting chunks of 36, until zero is reached, or there is a remainder

Long division HTU ÷ TU

972 ÷ 36

```
      27
36 ) 972
   - 720      20x36
     252
   - 252      7x36
       0
```

Answer : 27

Teach pupils to write a 'useful list' first at the side that will help them decide what chunks to use. Eg:

1 x = 36
10 x = 360
100x = 3600

Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots?' 'Can we use 100 lots?'

As children become more confident with the process, encourage more efficient chunks to get to the answer more quickly (eg: 20 x, 5 x) and expand on their 'useful lists'

Continue to develop different ways of presenting answers:

Long division

432 ÷ 15 becomes	432 ÷ 15 becomes	432 ÷ 15 becomes
1 5 | 4 3 2 = 28 r 12 3 0 0 1 3 2 1 2 0 1 2	1 5 | 4 3 2 = 28 3 0 0 (15×20) 1 3 2 1 2 0 (15×8) 1 2 $\frac{12}{15} = \frac{4}{5}$	1 5 | 4 3 2·0 = 28·8 3 0 ↓ 1 3 2 1 2 0 ↓ 1 2 0 1 2 0 0
Answer: 28 remainder 12	Answer: $28\frac{4}{5}$	Answer: 28·8

KEY VOCAB:

Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite numbers, common factor

Key SKILLS for division at Stage 6:

-Recall multiplication and division facts for all numbers up to 12 x 12 for more complex calculations
-Divide numbers up to 4 digits by a two digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
-Perform mental calculations, including with mixed operations and large numbers
-Identify common factors, common multiples and prime numbers
-Solve problems involving all 4 operations
-Use estimation to check answers to calculations and determine accuracy, in the context of a problem
-Use written division methods in cases where the answer has up to two decimal places
-Solve problems which require answers to be rounded to specified degrees of accuracy